



**APPLICATION OF CURVELET TRANSFORM AND PLC CONTROLLERS  
IN AUTOMATION AS ELEMENTS OF ENSURING HIGH-QUALITY CONTROL  
OF AN INDUSTRIAL GAS BURNER**

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**Abstract** – The paper describes and proposes a multidimensional curvelet transformation for flame image analysis and describes the use of a PLC controller in an original program for controlling an industrial gas burner mounted on metallurgical furnaces for burning metal molds. Both the use of the curvelet transform and automation using PLC controllers for controlling the gas burner are elements that ensure the accuracy and safety of the entire industrial process.

**Key words** – curvelet transform, gas burner, image processing, industrial control, PLC controller

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#### INTRODUCTION

Multidimensional transformations, and in particular time-frequency transformations such as continuous wavelet transform (CWT), ridglet or contourlet have found wide application in the analysis of images and measurement signals. In the case of wavelet transform, which has a uniform scaling law, large errors of image edge approximation occur during image processing [1].

The flame image processing process is a necessary operation to obtain data on the conducted process [2], [3]. The literature provides many ways to use the flame image processing path to identify the process and supervise the combustion process [4]. The curvelet transform, presented in the first part of the paper, introduces a novelty in the form of a non-uniform scaling law, designed specifically for the image edges, compared to the classical wavelet transform and increases the resolution in the time-frequency plane thanks to the orientation parameter [5], [6]. The transform was named curvelet 99 - after the date of 1999, when Candes and Donoho created a new multi-resolution transform [3]. Thanks to the orientation parameter, the curvelet transform has a higher time-frequency resolution compared to other multi-resolution transforms [7].

The project of automation of the process of controlling an industrial gas burner, which is part of a furnace for preheating molds in the glass industry, presented in the second part of this paper, was made in the LD language for the Siemens S7-1200 controller. The authors of this paper see the use of the curvelet transform [8], [9], [10], [11] as well as the automation of the process when controlling the gas burner using a PLC controller [12], [13], [14] to ensure the safety of the industrial process.

## 1 PROPERTIES OF THE CURVELET TRANSFORM AND ITS IMPLEMENTATION IN THE TIME DOMAIN

The basic operation in multiresolution transformations is bandpass filtering, i.e. separating high from low signal frequencies.

In the case of an image, high frequencies represent its edges [11].

The design of the low-pass filter  $P_0$  assumes low-frequency filtering  $|(\omega_1, \omega_1)| \leq 1$  - the set of high-pass filters  $\Delta_s$  for each scale  $s \geq 1$  filters high frequencies  $|(\omega_1, \omega_1)| \in [2^{2s}, 2^{2s+2}]$ . These filters fulfill Parseval's equality (1):

$$\|f\|_2^2 = \|P_0 f\|_2^2 + \sum_s \|\Delta_s f\|_2^2 \quad (1)$$

where:  $f$  - analyzed function;  $P_0$  - low-pass filter;  $\Delta_s$  - high-pass filter.

The curvelet transform coefficients for low frequencies are obtained from equation (2):

$$\alpha_\mu \equiv \langle \emptyset_{k_1, k_2}, P_0 f \rangle, \mu = (k_1, k_2) \in M' \quad (2)$$

where:  $\emptyset$  - scaling function

These coefficients are described in the region  $M'$ , and a single coefficient has coordinates  $(k_1, k_2)$ . For high frequencies, the individual coefficients of the curvelet transform are obtained based on equation (3), where  $\psi$  is an oriented ridgelet function, whose characteristic features are the parameters of orientation  $\theta$ , scale  $s$  and position  $(k_1, k_2)$ :

$$\alpha_\mu \equiv \langle \Delta_s, f, \psi_\mu \rangle, \mu \in M_s, s = 1, 2, \dots \quad (3)$$

where  $M_s$  is a specific single-scale dictionary that analyzes the selected, specific subband.

The definition of the curvelet transform can be written as (4):

$$\alpha_\mu \equiv \langle f, \gamma_\mu \rangle, \mu \in M' \quad (4)$$

where  $\gamma_\mu = \gamma_\mu(x_1, x_2)$  is an element made of  $\gamma_\mu = \Delta_s, f, \psi_\mu$ .

The transformation reconstruction is described by equation (5):

$$f = \sum_{\mu \in M'} \langle f, \gamma_\mu \rangle \gamma_\mu \quad (5)$$

The curvelet transform, unlike other multiresolution transforms, has an anisotropy scaling law.

This law allows for better approximation of image singularities such as edges.

The condition for creating a tight frame in the scaling process is to fulfill Parseval's law (6):

$$\|f\|_2^2 = \sum_{\mu \in M'} |\alpha_\mu|^2 \quad (6)$$

The non-uniform scaling law is satisfied when high frequencies (for  $s \geq 1$ ) are scaled according to the dependence  $|(\omega_1, \omega_2)| \in [2^{2s}, 2^{2s+2}]$  and at the same time the signal frequency domain is divided with respect to  $2^s$  orientation. This results in a different way of approximating the image edge using the curvelet transform than in the wavelet transform.

The algorithm implementing the curvelet transform can be divided into four main stages [15]:

1. Decomposition of a two-dimensional function  $f$  using filters  $P_0$  and  $\Delta_s$  where these are low-pass and high-pass filters, respectively (7):

$$f \rightarrow (P_0 f, \Delta_1 f, \Delta_2 f, \dots) \quad (7)$$

2. Passing the signal through the filter  $\Delta_s$  results in obtaining scale  $s$ . For each scale  $s$ , using a smooth window  $\bar{\omega}_Q$ , a division into dyadic areas  $Q$  is performed, where the number of these areas is determined by the scale  $s$  (8):

$$\Delta_s f \rightarrow (\bar{\omega}_Q \Delta_s f)_{Q \in Z_s} \quad (8)$$

3. Each of the obtained  $Q$  regions is renormalized to a unit value using the transfer and renormalization operator  $T_Q^{-1}$  (9):

$$g_Q = T_Q^{-1}(\bar{\omega}_Q \Delta_s f), Q \in Z_s \quad (9)$$

4. The curvelet transform coefficients are obtained from the ridgelet transform for each region.

Signal reconstruction can be presented in four steps:

1. By applying the inverse ridgelet transform, unit areas are obtained from the curvelet transform coefficients.

The unit areas obtained in this way are normalized to their original sizes – this is the inverse renormalization process (10):

$$h_Q = (T_Q)g_Q, Q \in Z_s \quad (10)$$

Using smooth windows  $\bar{\omega}_Q$  all areas for one scale  $s$  are integrated into one whole (11):

$$\Delta_s f = \sum_{Q \in Z_s} \bar{\omega}_Q h_Q \quad (11)$$

2. The original function  $f$  is reconstructed using the filters  $P_0$  and  $\Delta_s$  (12):

$$f = P_0(P_0 f) + \sum_{s \geq 0} \Delta_s(\Delta_s f). \quad (12)$$

The authors of this paper see the use of the above curvelet transform for digital processing of the image of a gas burner flame, in the system presented in the next chapter, which would allow the isolation of its characteristic features, used for a more accurate assessment of the combustion process.

## 2 PROCESS AUTOMATION USING A PLC CONTROLLER AS EXAMPLE OF A PROGRAM FOR CONTROLLING AN INDUSTRIAL GAS BURNER

The system presented in this chapter is used to automate the process of controlling an industrial gas burner, which is part of a furnace for preheating molds in the glass industry. The control program was created in LD language for the Siemens S7-1200 controller (Fig. 1).

The most important element of the entire control system is the PLC controller [16]. The direct influence on the choice of the controller was the number of its digital inputs (14) and outputs (9), which provides great possibilities for smaller solutions and programs without adding additional expansion modules. The advantage of this controller is that its environment allows for development through a wide selection of additional modules in the event of the expansion of the control or communication system.



Fig. 1. Siemens S7-1200 PLC controller with the symbol 6ES7215-1AG40-0XB0 [16]

The controller's power supply was selected as a dedicated Siemens power supply from the S7-1200 series, symbol 6EP1332-1SH71, which can be connected to a 120 or 230V AC network. The rest of the control system is powered by an Eaton power supply, symbol PSG240E, which can be connected to a network with a voltage of 100 to 240V AC. It provides a voltage of 24V DC at the output and can be loaded up to 10A, which is enough to control the relay coils. Additionally, an Omron temperature controller (Fig. 2) with the symbol E5CN-HR2M-500 was used to connect a thermocouple and facilitate temperature control, which is possible directly on the controller panel.

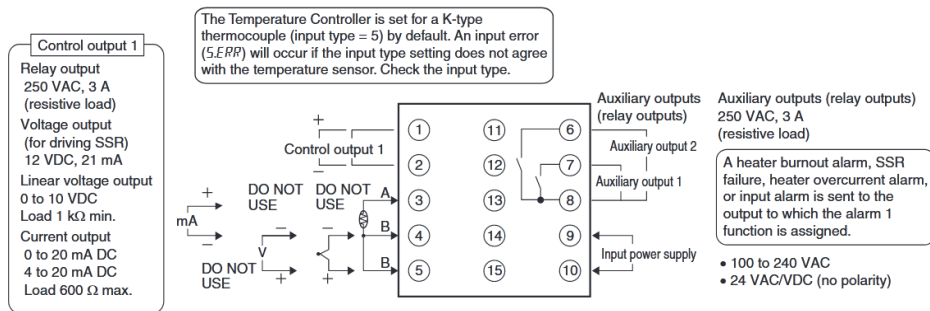


Fig. 2. Omron E5CN-H controller connection diagram [17]

A J-type thermocouple was used with this to measure the temperature inside the oven chamber. The RS PRO thermocouple part number 621-2394 has a top range of 700°C which is ideal for the temperature range of the preheating oven. The controller must be set correctly to work with the selected thermocouple. For a J-type thermocouple the range is -40°C to 700°C, the closest option on the controller is option 7 in the thermocouple type settings which has a range of -100°C to 850°C for a J-type thermocouple. The values of the selected thermocouple will be within the range supported by the controller for this option.

The controller program was implemented in a ladder diagram using the OpenPLC program (Fig. 3).

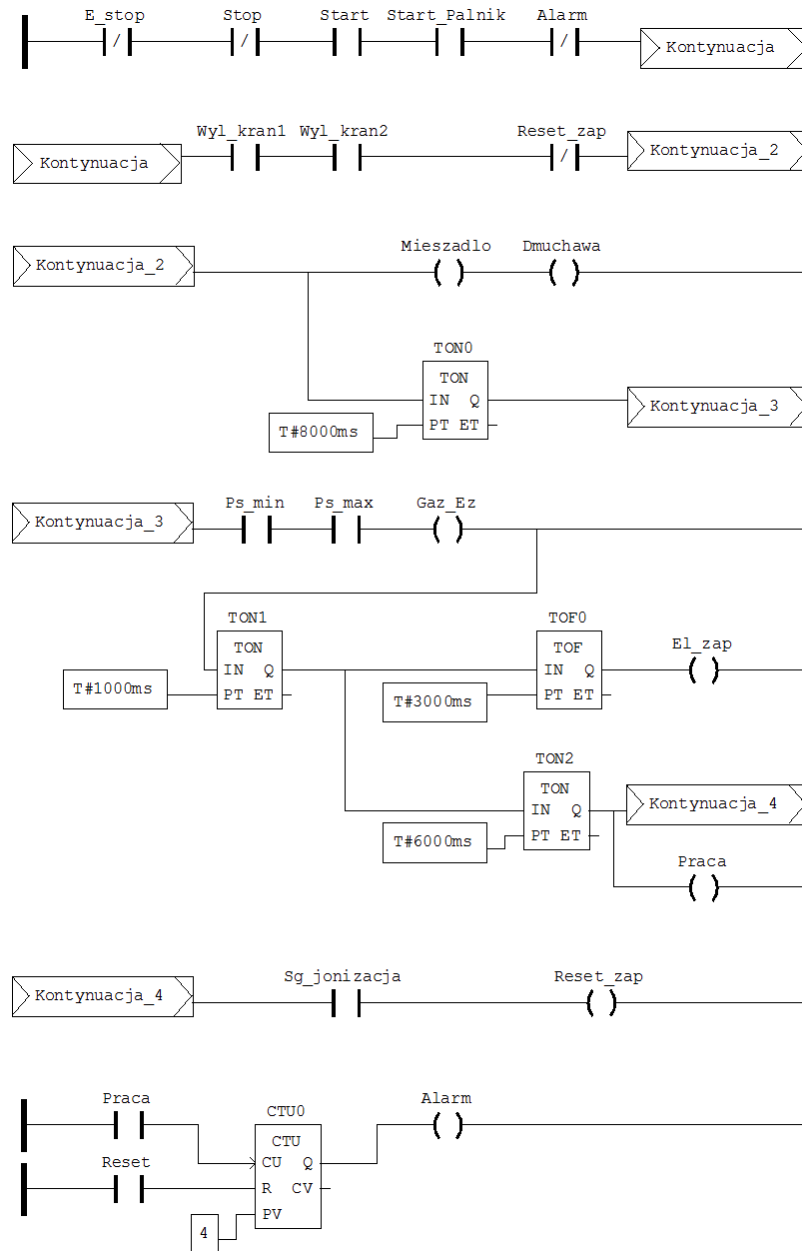


Fig. 3. PLC controller program in ladder diagram: 'Kontynuacja' – Continuation, 'Palnik' – Gas burner, 'Mieszadlo' – Mixer, 'Dmuchawa' – Blower, 'Praca' – Work, 'Jonizacja' – ionization [own research]

To check the correctness of the program functions, the PLC controller is virtually started, and then the program written for it. In OpenPLC, this is done by opening the debug mode and starting the PLC simulation. Both modes are marked in red (Fig. 4).

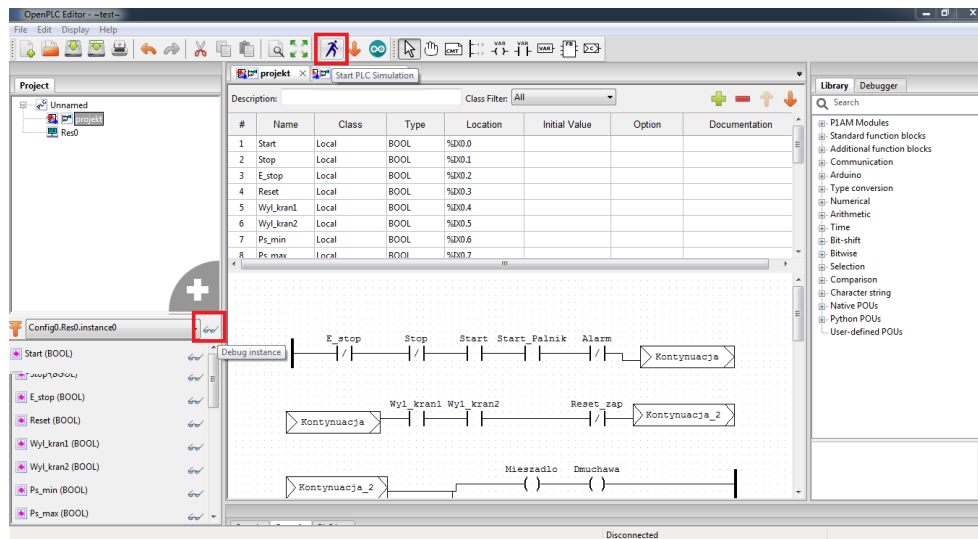
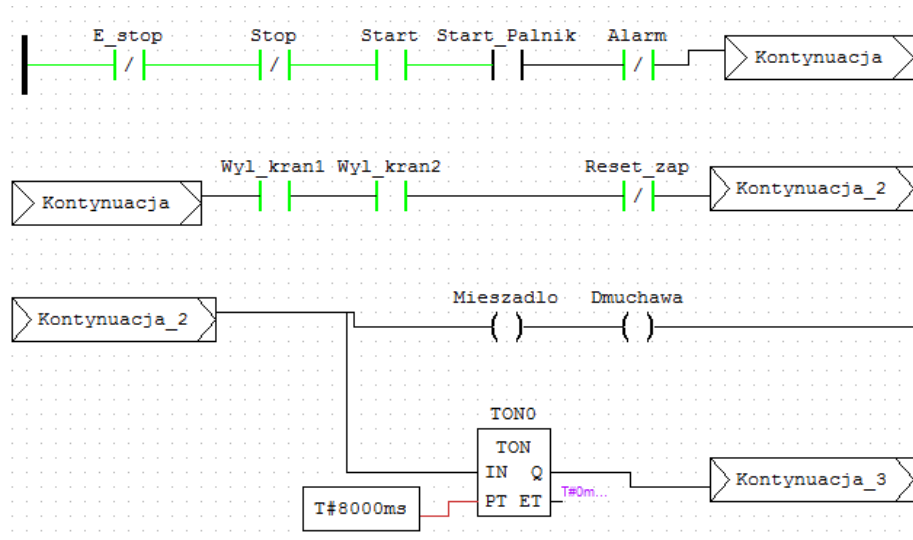


Fig. 4. OpenPLC Program Interface [own research]

To start the furnace control, the "Start" contact must be triggered, after which the next control phase can occur if the following conditions are met (Fig. 5):

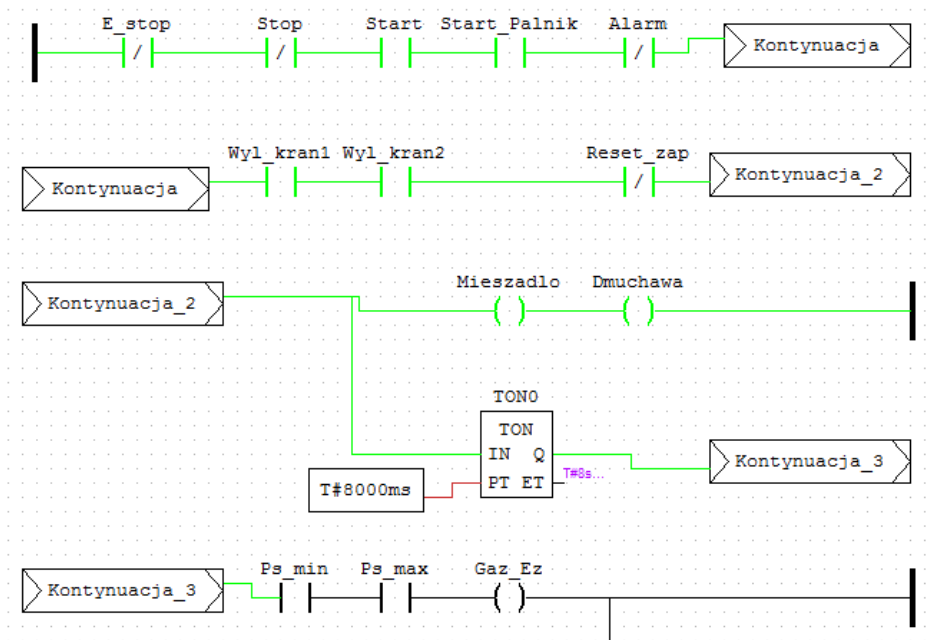
- The E\_stop emergency switch contact is not triggered,
- The Stop button contact is not triggered,
- The system is not in alarm mode and the Alarm contact is not triggered,
- The furnace door has been closed and the limit switch contacts on both sides have been triggered - Wyl\_kran1 and Wyl\_kran2,
- The ignition is not currently reset by triggering the Reset\_zap contact.



**Fig. 5. Part of the program during simulation with fulfilled operating conditions, before the signal to start the gas burner: 'Kontynuacja' – Continuation, 'Palnik' – Gas burner, 'Mieszadło' – Mixer, 'Dmuchawa' – Blower [own research]**

When the furnace is closed and the control system is ready, the Start\_burner contact is triggered by the temperature controller, which sends a signal when the temperature value is below the set value. Immediately after this, the coil of the mixer contactor on the furnace is triggered to circulate air and maintain similar temperatures throughout the furnace chamber. Then the coil of the blower contactor in the burner is activated to maintain a continuous air flow for correct operation of the burner or to clean it of gas residues after an unsuccessful attempt to ignite the burner. After such preparation, after 8 seconds counted by TON, a check is made whether the gas pressure supplied to the burner is within the minimum and maximum values specified by the pressure switches (Fig. 6).





**Fig. 6. Part of the program after the Start\_Palnik contact is triggered by the temperature controller, before the pressure switches operate correctly: 'Kontynuacja' – Continuation, 'Palnik' – Gas burner, 'Mieszadlo' – Mixer, 'Dmuchawa' – Blower [own research]**

If the gas pressure is within this range, then the Ps\_min and Ps\_max contacts, the gas solenoid valve coil is triggered. The gas flow to the combustion chambers is enabled and after 1 second counted by TON1, the voltage is supplied to the next stages. For 3 seconds counted by TOF0, an attempt is made to ignite the gas and air mixture in the combustion chamber. In parallel to the ignition attempt, TON2 counts down 6 seconds, and then the flame condition is checked. The Sg\_ionization contact is not triggered during correct ignition and does not supply voltage to the Reset\_zap coil responsible for making another attempt to ignite the burner (Fig. 7).

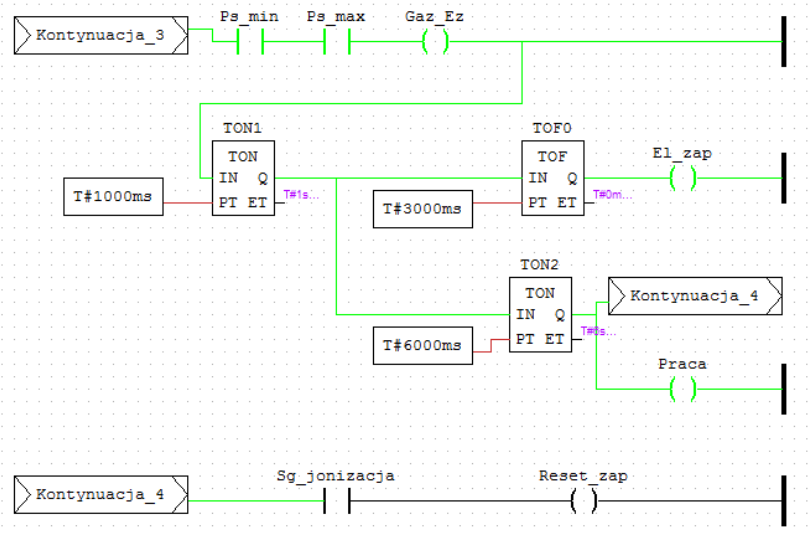


Fig. 7. Part of the program after checking the pressure switch conditions and starting the gas burner ignition process: 'Kontynuacja' – Continuation, 'Praca' – Work, 'Jonizacja' – ionization [own research]

After 6 seconds, the voltage triggers the "Praca" coil, which triggers the contact going into the CTU0 counter and adds 1 to the value in the counter (Fig. 8):

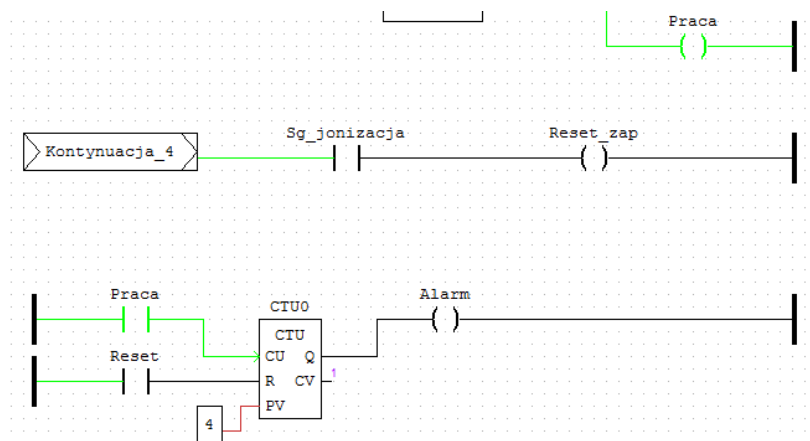


Fig. 8. Part of the program after triggering the "Praca" coil, which caused the CTU0 counter to count 1 visible at the CV output: 'Kontynuacja' – Continuation, 'Praca' – Work, 'Jonizacja' – ionization [own research]

After four unsuccessful ignition attempts, the counter triggers the "Alarm" coil, which in turn activates the NC Alarm contact, which turns off the control system (Fig. 9).

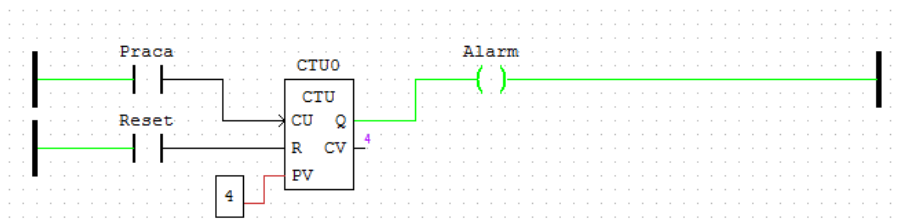


Fig. 9. Part of the program with alarm coil triggered, CV counter value has reached maximum PV value [own research]

Even though all conditions for correct operation of the control system are met, it is in alarm mode and it is impossible to repeat the burner start sequence without resetting the counter (Fig. 10).

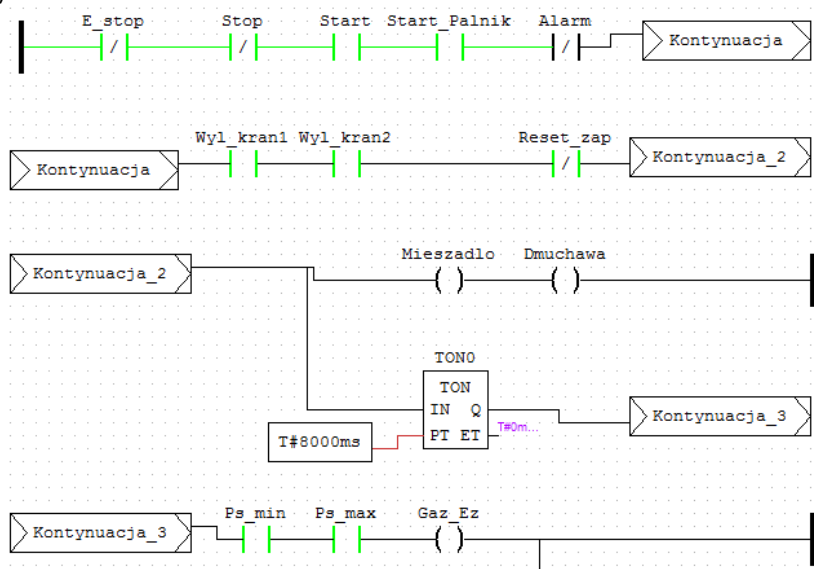


Fig. 10. Part of the program showing the control system in alarm mode preventing the gas burner from starting: 'Kontynuacja' – Continuation, 'Palnik' – Gas burner, 'Mieszadlo' – Mixer, 'Dmuchawa' – Blower [own research]

## CONCLUSIONS

The paper presents a mathematical tool for multi-resolution signal analysis, which is the curvelet transform, and describes the use of a PLC controller in an original program for controlling an industrial gas burner mounted on metallurgical furnaces for burning metal molds. The need to ensure safety when controlling an industrial gas burner generates the need for new methods of diagnosing and assessing the operation of a gas burner.

Based on previous scientific experience and observed features of the curvelet transform, the authors of this paper will attempt to use this transformation in the analysis of flame images recorded in laboratory and industrial conditions in the future. The authors note the fundamental advantage of the curvelet transform over other transformations in image discretization, based on the scale and orientation parameter, which can provide more information about the processed image.

The curvelet transform, as a mathematical tool designed for image analysis, taking into account its directional features, seems to be a natural solution for flame image analysis. By using a PLC controller in the control system, unlike the old contactor control systems, the operation of the system can be simplified and improved, which increases its safety. Detection, identification and resolution of errors is then much easier thanks to the possibility of checking the status of the controller's outputs and inputs or direct connection to the controller to read errors.

The presented program works correctly, goes through subsequent programmed stages and responds to errors in an appropriate manner - by triggering an alarm and stopping the operation of components in a safe manner. By using a temperature controller with an integrated operator panel, connected directly to the PLC controller, it is possible to easily adjust the temperature settings by the machine operator, without reprogramming the settings in the PLC controller program each time when it would be required. If it is necessary to change or expand the control system, it is possible, without replacing all the elements, such possibilities are provided by additional modules and reprogramming the controller.

In the future, it is possible to add network functions and connect the furnace and burner to a larger control structure or connect several such furnaces for heating molds to work in parallel. In the case of a flame image obtained from an optical monitoring system, changes in its shape or position are clearly related to changes in the combustion process regime. The nature of these changes can be considered in many areas, e.g. changes caused by input factors or a failure state caused by, for example, the flame breaking away from the burner crown or the flame disappearing. Therefore, the possibility of using the curvelet transform, or more precisely, transforming the flame image into the form of mathematical coefficients describing the image, seems desirable and expected from the point of view of the accuracy of the analysis of this image.

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