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Description Of A Three-Phase, Four-Wire System With A Nonlinear Receiver And An Asymmetric Voltage Source Using CPC Power Theory

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Abstract – This article mathematically describes a three-phase, four-wire circuit in the case of a nonlinear, unbalanced load, asymmetry of the power source with a periodic, non-sinusoidal waveform. This description uses Currents' Physical Components (CPC) power theory for three-phase circuits. Determining the energy flow between the source and the load is possible by decomposing the phase current into components depending on the physical nature of the phenomena in this circuit. Mathematical relationships were determined enabling decomposition into components depending on the direction of energy flow and the causes of their creation. A calculation example using the determined relationships and calculation results has been presented. The presented computational concept is important for mathematical analyzes in circuits with nonlinear three-phase receivers. Knowing the nature of physical phenomena, it is possible to perform measures that limit the value of the current supplying the load.

Keywords – Currents Physical Components, nonlinear receiver, power theory

Introduction

The problem of mathematical description of the energy flow between the source and the receiver is widely discussed in the literature [1]-[18] and several power theories have been developed. These theories take a different approach and in practice are analyzed for electrical circuits with certain limitations. The most promising are two theories: Emanuel's theory [8] and Czarnecki's CPC [3]. These are two different approaches that have their applications in practical solutions. Emanuel's theory has found applications in economic settlements and measurements in the power system, while the CPC theory works well in the case of detailed mathematical analyzes and power factor improvement.

Active power for a passive system has been precisely described by the active component of the current according to Fryze's theory [1], while the definition of apparent power in three-phase circuits is ambiguous.

There is a well-known definition of apparent power by F.Bucholz [2] from 1922 for a three-phase source with sinusoidal waveforms in the form:

$$S = \sqrt{U_{\mathrm{R}}^2 + U_{\mathrm{S}}^2 + U_{\mathrm{T}}^2} \cdot \sqrt{I_{\mathrm{R}}^2 + I_{\mathrm{S}}^2 + I_{\mathrm{T}}^2}\,. \tag{1}$$

The apparent power presented in this form has not been widely used in technology, but in special cases it is still used today. The form of apparent power introduced in 1935 by H.L.Curtis and F.B.Silsbee, called arithmetic apparent power, became more popular:

$$\underline{S} = \underline{U}_{\mathrm{R}} \underline{I}_{\mathrm{R}}^{*} + \underline{U}_{\mathrm{S}} \underline{I}_{\mathrm{S}}^{*} + \underline{U}_{\mathrm{T}} \underline{I}_{\mathrm{T}}^{*} = S \cdot e^{j\varphi}\,. \tag{2}$$

The third definition of this power is a commonly used form called geometric apparent power:

$$S = \sqrt{P^2 + Q^2}\,. \tag{3}$$

Dependencies (1), (2) and (3) give the same results only in the case of symmetry, for a three-phase balanced receiver, with sinusoidal current and voltage waveforms. Failure to meet even one of these conditions causes the results obtained from these equations to be contradictory.

Correct performance of power compensation [11], [12], [19], [20], as well as determining the energy parameters of three-phase circuits depend on the adopted definition of apparent power. Building the power theory on several foundations of apparent power causes the obtained results to be contradictory even in the case of sinusoidal waveforms.

The power factor is closely related to energy losses in the energy system [21]-[40]. Lowering the power factor value increases transmission losses. It is therefore proposed to choose the definition of apparent power, paying particular attention to the assessment of these losses. Only the apparent power calculated according to equation (1) gives the correct power factor value. This definition is an extension of the definition in single-phase circuits $S = \|u\| \cdot \|i\|$.

1 MATHEMATICAL DESCRIPTION OF THE CIRCUIT

The aim of the analysis is to decompose the current into components depending on the physical properties in the circuit. Such decomposition will enable the construction of a method to improve the power factor in a situation where the receiver is a nonlinear three-phase, four-wire, unbalanced system, powered by a three-phase source with non-sinusoidal periodic waveforms.

Harmonics of voltages with positive and negative sequences are transmitted through the transformer (Fig. 1).

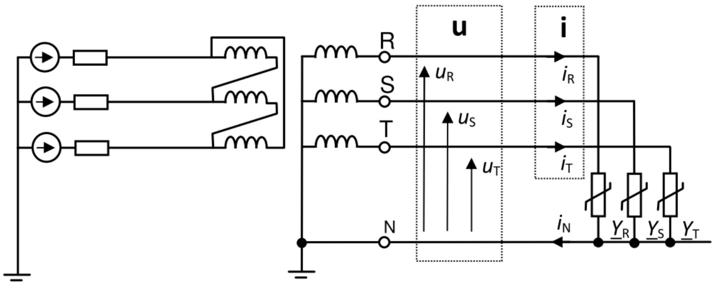

Fig. 1. Power system structure for a nonlinear three-phase receiver in Y configuration

Symmetrical harmonics of zero sequences are not transferred from the source to the secondary side of the transformer. Such harmonics can only occur on the secondary side of the transformer if they are generated by a nonlinear load. The three-phase voltage and current vectors are defined as follows:

$$\mathbf{u} = \sum_n \mathbf{u}_n = \sqrt{2} \cdot \Re e \sum_n \underline{\mathbf{U}}_n \cdot e^{jn\omega_1 t}, \quad \mathbf{u}_n = \begin{bmatrix} u_{\mathrm{R}n} \\ u_{\mathrm{S}n} \\ u_{\mathrm{T}n} \end{bmatrix}, \tag{4}$$

$$\mathbf{i} = \sum_n \mathbf{i}_n = \sqrt{2} \cdot \Re e \sum_n \underline{\mathbf{I}}_n \cdot e^{jn\omega_1 t}, \quad \mathbf{i}_n = \begin{bmatrix} i_{\mathrm{R}n} \\ i_{\mathrm{S}n} \\ i_{\mathrm{T}n} \end{bmatrix}, \tag{5}$$

where the complex three-phase vectors are:

$$\underline{\mathbf{U}}_n = \begin{bmatrix} \underline{U}_{\mathrm{R}n} \\ \underline{U}_{\mathrm{S}n} \\ \underline{U}_{\mathrm{T}n} \end{bmatrix}, \qquad \underline{\mathbf{I}}_n = \begin{bmatrix} \underline{I}_{\mathrm{R}n} \\ \underline{I}_{\mathrm{S}n} \\ \underline{I}_{\mathrm{T}n} \end{bmatrix}. \tag{6}$$

The effective (RMS) values of three-phase voltage and current are:

$$\|\mathbf{u}\| = \sqrt{\sum_{n=1}^{N} \left(\|u_{\mathrm{R}n}\|^2 + \|u_{\mathrm{S}n}\|^2 + \|u_{\mathrm{T}n}\|^2 \right)}, \tag{7}$$

$$\|\mathbf{i}\| = \sqrt{\sum_{n=1}^{N} \left(\|i_{\mathrm{R}n}\|^2 + \|i_{\mathrm{S}n}\|^2 + \|i_{\mathrm{T}n}\|^2 \right)}. \tag{8}$$

The nonlinear receiver behaves as a harmonic generator and a load on the power grid. Energy phenomena associating the generation of harmonics by a nonlinear receiver will be marked with the index "*C*", while those having their source in the power system will be marked with the index "*D*".

The description of power and current distributions according to the CPC theory will be possible by observing the instantaneous values of line currents i_R, i_S, i_T and voltages u_R, u_S, u_T relative to the neutral point *N*. In the case of non-sinusoidal periodic waveforms, the direction of energy flow between the source and the nonlinear receiver may be different for individual harmonics [6]. It will depend on which of the *C* or *D* systems contributes to the creation of a given harmonic. It is best to consider the situation for each n^{th} harmonic separately. The set of all harmonics occurring in the system (Fig. 2) is denoted by "*N*". This set can be divided into: the set of harmonics N_D corresponding to the pulsations of the components describing the energy flow from the *D* to *C* system, and N_C - the opposite flow.

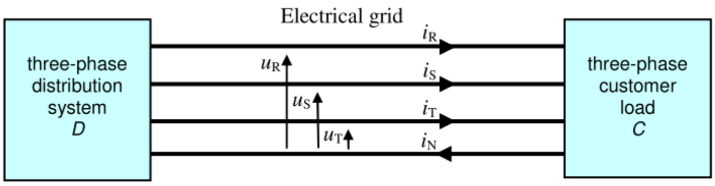

Fig. 2 Combination of a nonlinear receiver (*C* system) and a three-phase source (*D* system)

The active power associated with the flow of energy from the source to the receiver is the positive power P_{Dn}, and the negative power associated with the opposite direction is P_{Cn}. The direction of energy transfer in a three-phase circuit is determined by examining the sign of the three-phase active power of the n^{th} harmonic P_n.

$$P_n = U_{Rn} I_{Rn} \cos\varphi_{Rn} + U_{Sn} I_{Sn} \cos\varphi_{Sn} + U_{Tn} I_{Tn} \cos\varphi_{Tn}. \tag{9}$$

This approach assumes that the direction of energy flow for the n^{th} harmonic in all phases is the same. If the directions of energy flow in individual phases are different, the three-phase active power of the n^{th} harmonic P_n will be the resultant power characterizing the three-phase power circuit.

With the adopted notation, the set of *N* harmonics was divided into N_D and N_C sets [19].

$$\left.\begin{array}{l} P_n \geq 0 \\ P_n < 0 \end{array}\right\} \begin{array}{ll} P_{Dn} = P_n, & n \in N_D \\ P_{Cn} = -P_n, & n \in N_C \end{array} \tag{10}$$

When the active power P_n for the n^{th} harmonic is positive, it should be assumed that it comes from the *D* system (Fig. 3a), or the power produced by it P_{Dn} dominates, due to the direction of energy flow, over the harmonic current of the same order generated in P_{Cn} receiver. Otherwise, it is possible to extract a set of N_C harmonics (Fig. 3b).

Active power is the sum of two powers [9]:

- P_D – **the active operating power** - generated by the voltage source E_D and lost on the admittance of the Y_C receiver (Fig. 3a), and
- P_C – **the active reflected power** - generated by the current source J_C and lost on the internal impedance Z_D of the source (Fig. 3b).

The P_D component is positive - due to the flow of energy from the source to the receiver, while P_C is negative - due to the change in the sign of the current direction.

The set of N harmonics contains subsets N_D and N_C, which group harmonics in terms of the direction of energy flow of the n^{th} harmonic (n = 1...N) from D to C and from C to D, respectively.

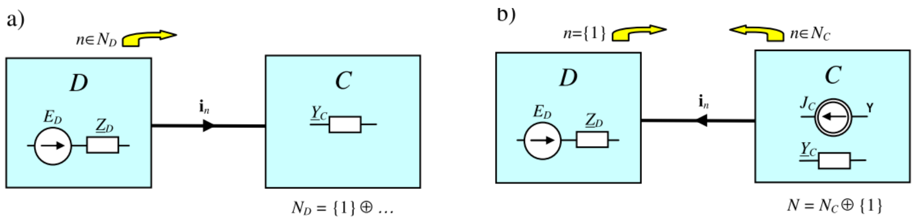

Fig. 3 Block representation of the concept of energy flow between *C* and *D* systems

In certain situations, it is possible to observe the generation of the same harmonics by a nonlinear receiver that already existed in the power grid. In practical measurements, this condition is difficult to locate and is manifested by an increase in the amplitude of the n^{th} voltage harmonic after connecting a nonlinear receiver. In this situation, energy flow is possible either in the direction from D to C or from C to D.

$$\mathbf{i}_D = \sum_{n \in N_D} \mathbf{i}_n, \quad \mathbf{u}_D = \sum_{n \in N_D} \mathbf{u}_n, \quad P_D = \sum_{n \in N_D} P_n, \tag{11}$$

while with the opposite flow they are:

$$\mathbf{i}_C = \sum_{n \in N_C} \mathbf{i}_n, \quad \mathbf{u}_C = -\sum_{n \in N_C} \mathbf{u}_n, \quad P_C = -\sum_{n \in N_C} P_n. \tag{12}$$

In this way, the components were decomposed into two subsets related to the direction of energy flow, which is a consequence of the physical phenomenon that caused it:

$$\mathbf{i} = \mathbf{i}_D + \mathbf{i}_C, \quad \mathbf{u} = \mathbf{u}_D - \mathbf{u}_C. \tag{13}$$

Due to the fact that the N_C and N_D sets are disjoint, the voltage and current components are mutually orthogonal, so the following relationships occur:

$$\|\mathbf{i}\|^2 = \|\mathbf{i}_D\|^2 + \|\mathbf{i}_C\|^2, \quad \|\mathbf{u}\|^2 = \|\mathbf{u}_D\|^2 + \|\mathbf{u}_C\|^2. \tag{14}$$

The active power, together with the current and voltage at the load terminals, were divided into components related to the direction of energy flow.

The interaction between source and load is considered through the superposition of both phenomena C and D analyzed individually.

The energy flow from the source to the load is analyzed for harmonics belonging to the N_D set. In this direction of energy, the current must be decomposed into components that are physically dependent on individual phenomena in the circuit - in accordance with the concept of the CPC power theory.

The first and most important component is the element related to active power. The equivalent conductance in a four-wire system for the n^{th} harmonic is:

$$G_{en} = \frac{P_{Dn}}{\|\mathbf{u}_{Dn}\|^2} = \frac{1}{m}\left(G_{\mathrm{R}n} + G_{\mathrm{S}n} + G_{\mathrm{T}n}\right), \tag{15}$$

where: m – multiple of active phases.

This conductance is determined from the active power P_D affecting the phase conductances G_R, G_S, G_T of the C system. The equivalent conductance is associated with the current component, which in the case of a non-sinusoidal periodic waveform is:

$$\mathbf{i}_{\mathrm{c}Dn} = G_{\mathrm{e}n} \cdot \mathbf{u}_{Dn} = \sqrt{2}\Re e\left\{G_{\mathrm{e}n} \cdot \underline{\mathbf{U}}_{Dn} \cdot e^{jn\omega_1 t}\right\}. \tag{16}$$

Therefore, the equivalent conductance to the n^{th} harmonic is associated with the component $\mathbf{i}_{cD}$, the resultant value of which is:

$$\mathbf{i}_{\mathrm{c}D} = \sum_{n\in N_D} \mathbf{i}_{\mathrm{c}n} = \sqrt{2}\Re e \sum_{n\in N_D} G_{\mathrm{e}n} \underline{\mathbf{U}}_n e^{jn\omega_1 t}. \tag{17}$$

The current component $\mathbf{i}_{aD}$, independent of the harmonic, depends on the resultant active power P_D and the value of the equivalent conductance G_e.

$$\mathbf{i}_{\mathrm{a}D} = \sqrt{2}\Re e\left\{G_\mathrm{e} \cdot \underline{\mathbf{U}}_D \cdot e^{j\omega t}\right\} \tag{18}$$

The value of the conductance G_e is determined knowing the resultant active power P_D of the three-phase load from the relationship:

$$G_\mathrm{e} = \frac{P_D}{\|\mathbf{u}_D\|^2} = \frac{\|\mathbf{i}_{\mathrm{a}D}\|}{\|\mathbf{u}_D\|}. \tag{19}$$

When the conductance $G_{\mathrm{e}n}$ varies with the order of the harmonic, there is a non-zero scattered component of current $\mathbf{i}_{sD}$. This component is equal:

$$\mathbf{i}_{\mathrm{s}D} = \mathbf{i}_{\mathrm{c}D} - \mathbf{i}_{\mathrm{a}D} = \sqrt{2}\Re e \sum_{n\in N_D} G_{\mathrm{e}n} \underline{\mathbf{U}}_n e^{jn\omega_1 t} - \sqrt{2}\Re e \sum_{n\in N_D} G_\mathrm{e} \underline{\mathbf{U}}_n e^{jn\omega_1 t} = \sqrt{2}\Re e \sum_{n\in N_D} \left(G_{\mathrm{e}n} - G_\mathrm{e}\right) \underline{\mathbf{U}}_n e^{jn\omega_1 t}. \tag{20}$$

The effective value of the scattered component is:

$$\left\|\mathbf{i}_{sD}\right\| = \sqrt{\sum_{n\in N_D}\left(G_{en} - G_{e}\right)^2 \left\|\mathbf{u}_n\right\|^2}\,. \tag{21}$$

The shift of the current relative to voltage function is obtained by introducing the equivalent susceptance, which is:

$$B_{en} = -\frac{Q_{Dn}}{\left\|\mathbf{u}_{Dn}\right\|^2} = \frac{1}{m}\left(B_{Rn} + B_{Sn} + B_{Tn}\right). \tag{22}$$

Equations (15) and (22) give the formula for the equivalent admittance in a four-wire system:

$$\underline{Y}_{en} = G_{en} + jB_{en} = \frac{1}{m}\left(\underline{Y}_{Rn} + \underline{Y}_{Sn} + \underline{Y}_{Tn}\right). \tag{23}$$

Using the complex rotation factor:

$$\alpha = 1e^{j\frac{2\pi}{3}}, \quad \alpha^* = 1e^{-j\frac{2\pi}{3}}$$

it is possible to define a three-phase unit vector for the positive $\mathbf{1}^{\mathrm{p}}$, negative $\mathbf{1}^{\mathrm{n}}$ and zero $\mathbf{1}^{\mathrm{z}}$ sequences:

$$\mathbf{1}^{\mathrm{p}} = \begin{bmatrix} 1 \\ \alpha^* \\ \alpha \end{bmatrix}, \quad \mathbf{1}^{\mathrm{n}} = \begin{bmatrix} 1 \\ \alpha \\ \alpha^* \end{bmatrix}, \quad \mathbf{1}^{\mathrm{z}} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}. \tag{24}$$

In a four-wire system, in the case of asymmetry, the phase voltages for the n^{th} harmonic are described by symmetrical components, which can be determined from the relationship:

$$\begin{bmatrix} \underline{U}_n^{\mathrm{z}} \\ \underline{U}_n^{\mathrm{p}} \\ \underline{U}_n^{\mathrm{n}} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^* \\ 1 & \alpha^* & \alpha \end{bmatrix} \cdot \begin{bmatrix} \underline{U}_{Rn} \\ \underline{U}_{Sn} \\ \underline{U}_{Tn} \end{bmatrix}, \qquad \begin{bmatrix} \underline{U}_{Rn} \\ \underline{U}_{Sn} \\ \underline{U}_{Tn} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha^* & \alpha \\ 1 & \alpha & \alpha^* \end{bmatrix} \cdot \begin{bmatrix} \underline{U}_n^{\mathrm{z}} \\ \underline{U}_n^{\mathrm{p}} \\ \underline{U}_n^{\mathrm{n}} \end{bmatrix}. \tag{25}$$

The equivalent susceptance (22) is associated with the reactive component of current, which is consistent with the phase sequence of the power source and is equal:

$$\mathbf{i}_{rDn} = \sqrt{2}\Re e\left\{jB_{en} \cdot \underline{\mathbf{U}}_{Dn} \cdot e^{jn\omega_1 t}\right\}, \quad \left\|\mathbf{i}_{rD}\right\| = \sqrt{\sum_{n\in N_D} B_{en}^2 \left\|\mathbf{u}_n\right\|^2}\,. \tag{26}$$

When the load is unbalanced, the n^{th} harmonic of the current also contains an unbalanced current, which is equal:

$$\mathbf{i}_{\mathrm{u}Dn} = \mathbf{i}_{Dn} - \mathbf{i}_{\mathrm{c}Dn} - \mathbf{i}_{\mathrm{r}Dn} = \sqrt{2}\Re e\left\{\mathbf{i}_{Dn} - \underline{Y}_{\mathrm{e}n}\left(\mathbf{1}^{\mathrm{p}}\underline{U}^{\mathrm{p}}_{Dn} + \mathbf{1}^{\mathrm{n}}\underline{U}^{\mathrm{n}}_{Dn} + \mathbf{1}^{\mathrm{z}}\underline{U}^{\mathrm{z}}_{Dn}\right)\cdot e^{jn\omega_1 t}\right\}. \tag{27}$$

The unbalanced component (27) is asymmetric, which can be represented as the sum of vectors of symmetric components with positive, negative and zero sequences:

$$\mathbf{i}_{\mathrm{u}Dn} = \mathbf{i}^{\mathrm{p}}_{\mathrm{u}Dn} + \mathbf{i}^{\mathrm{n}}_{\mathrm{u}Dn} + \mathbf{i}^{\mathrm{z}}_{\mathrm{u}Dn}. \tag{28}$$

This current can be expressed in quantities describing a three-phase load. For a symmetric component on positive sequence it will be:

$$\mathbf{i}^{\mathrm{p}}_{\mathrm{u}Dn} = \sqrt{2}\Re e\left\{\begin{bmatrix}(\underline{Y}_{\mathrm{R}n} - Y_{en})\\(\underline{Y}_{\mathrm{S}n} - Y_{en})\alpha^*\\(\underline{Y}_{\mathrm{T}n} - Y_{en})\alpha\end{bmatrix}\cdot \underline{U}^{\mathrm{p}}_{Dn}e^{jn\omega_1 t}\right\}, \tag{29}$$

while for a symmetric component on negative sequence it will be:

$$\mathbf{i}^{\mathrm{n}}_{\mathrm{u}Dn} = \sqrt{2}\Re e\left\{\begin{bmatrix}(\underline{Y}_{\mathrm{R}n} - Y_{en})\\(\underline{Y}_{\mathrm{S}n} - Y_{en})\alpha\\(\underline{Y}_{\mathrm{T}n} - Y_{en})\alpha^*\end{bmatrix}\cdot \underline{U}^{\mathrm{n}}_{Dn}e^{jn\omega_1 t}\right\}, \tag{30}$$

for a symmetric component on zero sequence it will be:

$$\mathbf{i}^{\mathrm{z}}_{\mathrm{u}Dn} = \sqrt{2}\Re e\left\{\begin{bmatrix}\underline{Y}_{\mathrm{R}n} - Y_{en}\\\underline{Y}_{\mathrm{S}n} - Y_{en}\\\underline{Y}_{\mathrm{T}n} - Y_{en}\end{bmatrix}\cdot \underline{U}^{\mathrm{z}}_{Dn}e^{jn\omega_1 t}\right\}. \tag{31}$$

These vectors are mutually orthogonal, so for effective values it is correct to write:

$$\|\mathbf{i}_{\mathrm{u}Dn}\|^2 = \|\mathbf{i}^{\mathrm{p}}_{\mathrm{u}Dn}\|^2 + \|\mathbf{i}^{\mathrm{n}}_{\mathrm{u}Dn}\|^2 + \|\mathbf{i}^{\mathrm{z}}_{\mathrm{u}Dn}\|^2. \tag{32}$$

The complex RMS values of the unbalanced component create the relationship:

$$\begin{bmatrix}\underline{I}^{\mathrm{z}}_{\mathrm{u}Dn}\\\underline{I}^{\mathrm{p}}_{\mathrm{u}Dn}\\\underline{I}^{\mathrm{n}}_{\mathrm{u}Dn}\end{bmatrix} = \frac{1}{3}\begin{bmatrix}1 & 1 & 1\\1 & \alpha & \alpha^*\\1 & \alpha^* & \alpha\end{bmatrix}\cdot\begin{bmatrix}\underline{I}_{\mathrm{Ru}n}\\\underline{I}_{\mathrm{Su}n}\\\underline{I}_{\mathrm{Tu}n}\end{bmatrix}. \tag{33}$$

This means that for harmonics in the positive sequence, according to (29), from equality (33) one obtains:

$$\begin{bmatrix} \underline{I}^{\mathrm{z}}_{\mathrm{u}Dn} \\ \underline{I}^{\mathrm{p}}_{\mathrm{u}Dn} \\ \underline{I}^{\mathrm{n}}_{\mathrm{u}Dn} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^* \\ 1 & \alpha^* & \alpha \end{bmatrix} \cdot \begin{bmatrix} (\underline{Y}_{\mathrm{R}n} - \underline{Y}_{en}) \\ (\underline{Y}_{\mathrm{S}n} - \underline{Y}_{en})\alpha^* \\ (\underline{Y}_{\mathrm{T}n} - \underline{Y}_{en})\alpha \end{bmatrix} \cdot \underline{U}^{\mathrm{p}}_{Dn} = \frac{1}{3}\begin{bmatrix} \underline{Y}_{\mathrm{R}n} + \alpha^*\underline{Y}_{\mathrm{S}n} + \alpha\underline{Y}_{\mathrm{T}n} \\ 0 \\ \underline{Y}_{\mathrm{R}n} + \alpha\underline{Y}_{\mathrm{S}n} + \alpha^*\underline{Y}_{\mathrm{T}n} \end{bmatrix} \cdot \underline{U}^{\mathrm{p}}_{Dn} = \begin{bmatrix} \underline{Y}^{\mathrm{z}}_{\mathrm{u}n} \\ 0 \\ \underline{Y}^{\mathrm{n}}_{\mathrm{u}n} \end{bmatrix} \cdot \underline{U}^{\mathrm{p}}_{Dn} \quad (34)$$

where $\underline{Y}_{un}$ – the unbalanced admittance.

For harmonics in the negative sequence, according to (30), from equality (33) one obtains:

$$\begin{bmatrix} \underline{I}^{\mathrm{z}}_{\mathrm{u}Dn} \\ \underline{I}^{\mathrm{p}}_{\mathrm{u}Dn} \\ \underline{I}^{\mathrm{n}}_{\mathrm{u}Dn} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^* \\ 1 & \alpha^* & \alpha \end{bmatrix} \cdot \begin{bmatrix} (\underline{Y}_{\mathrm{R}n} - \underline{Y}_{en}) \\ (\underline{Y}_{\mathrm{S}n} - \underline{Y}_{en})\alpha \\ (\underline{Y}_{\mathrm{T}n} - \underline{Y}_{en})\alpha^* \end{bmatrix} \cdot \underline{U}^{\mathrm{n}}_{Dn} = \frac{1}{3}\begin{bmatrix} \underline{Y}_{\mathrm{R}n} + \alpha\underline{Y}_{\mathrm{S}n} + \alpha^*\underline{Y}_{\mathrm{T}n} \\ \underline{Y}_{\mathrm{R}n} + \alpha^*\underline{Y}_{\mathrm{S}n} + \alpha\underline{Y}_{\mathrm{T}n} \\ 0 \end{bmatrix} \cdot \underline{U}^{\mathrm{n}}_{Dn} = \begin{bmatrix} \underline{Y}^{\mathrm{z}}_{\mathrm{n}n} \\ \underline{Y}^{\mathrm{p}}_{\mathrm{n}n} \\ 0 \end{bmatrix} \cdot \underline{U}^{\mathrm{n}}_{Dn} \quad (35)$$

while for harmonics in the zero sequence, according to (31), from equality (33) one obtains:

$$\begin{bmatrix} \underline{I}^{\mathrm{z}}_{\mathrm{u}Dn} \\ \underline{I}^{\mathrm{p}}_{\mathrm{u}Dn} \\ \underline{I}^{\mathrm{n}}_{\mathrm{u}Dn} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^* \\ 1 & \alpha^* & \alpha \end{bmatrix} \cdot \begin{bmatrix} \underline{Y}_{\mathrm{R}n} - \underline{Y}_{en} \\ \underline{Y}_{\mathrm{S}n} - \underline{Y}_{en} \\ \underline{Y}_{\mathrm{T}n} - \underline{Y}_{en} \end{bmatrix} \cdot \underline{U}^{\mathrm{z}}_{Dn} = \frac{1}{3}\begin{bmatrix} 0 \\ \underline{Y}_{\mathrm{R}n} + \alpha\underline{Y}_{\mathrm{S}n} + \alpha^*\underline{Y}_{\mathrm{T}n} \\ \underline{Y}_{\mathrm{R}n} + \alpha^*\underline{Y}_{\mathrm{S}n} + \alpha\underline{Y}_{\mathrm{T}n} \end{bmatrix} \cdot \underline{U}^{\mathrm{z}}_{Dn} = \begin{bmatrix} 0 \\ \underline{Y}^{\mathrm{p}}_{\mathrm{u}n} \\ \underline{Y}^{\mathrm{n}}_{\mathrm{u}n} \end{bmatrix} \cdot \underline{U}^{\mathrm{z}}_{Dn}. \quad (36)$$

Comparing equations (34), (35) and (36), an complication can be observed in the definition of the unbalanced admittance $\underline{Y}_{un}$. In order to standardize the notation, a generalized complex rotation coefficient can be used:

$$\beta_{(n)} = (\alpha^*)^n = 1e^{-j\frac{2\pi \cdot n}{3}} = \begin{cases} 1, & \text{for } n = 3k+3 \\ \alpha^*, & \text{for } n = 3k+1 \\ \alpha, & \text{for } n = 3k+2 \end{cases} \quad (37)$$

The unbalanced admittance decomposed into symmetric components in the universal notation will be in the form:

$$\underline{Y}^{\mathrm{z}}_{\mathrm{u}n} = \frac{1}{3}\left[\left(\underline{Y}_{\mathrm{R}n} + \beta_{(n)}\underline{Y}_{\mathrm{S}n} + \beta^*_{(n)}\underline{Y}_{\mathrm{T}n}\right) - \underline{Y}_{en}\left(1 + \beta_{(n)} + \beta^*_{(n)}\right)\right], \quad (38)$$

$$\underline{Y}^{\mathrm{p}}_{\mathrm{u}n} = \frac{1}{3}\left[\left(\underline{Y}_{\mathrm{R}n} + \alpha\beta_{(n)}\underline{Y}_{\mathrm{S}n} + \alpha^*\beta^*_{(n)}\underline{Y}_{\mathrm{T}n}\right) - \underline{Y}_{en}\left(1 + \alpha\beta_{(n)} + \alpha^*\beta^*_{(n)}\right)\right], \quad (39)$$

$$\underline{Y}^{\mathrm{n}}_{\mathrm{u}n} = \frac{1}{3}\left[\left(\underline{Y}_{\mathrm{R}n} + \alpha^*\beta_{(n)}\underline{Y}_{\mathrm{S}n} + \alpha\beta^*_{(n)}\underline{Y}_{\mathrm{T}n}\right) - \underline{Y}_{en}\left(1 + \alpha^*\beta_{(n)} + \alpha\beta^*_{(n)}\right)\right] \quad (40)$$

The values of these admittances depend not only on the load admittance for the n^{th} harmonic, but also on the harmonic sequence. Moreover, analyzing equations (34), (35) and (36), it can be seen that for the n^{th} harmonic only two unbalanced admittances have a non-zero value. The third takes a value equal to zero.

Multiplications of the rotation coefficients α and $\beta_{(n)}$ cause an angular jump of the rotation vector by 120^0 according to the relationship:

$$\begin{gathered}\alpha \cdot \beta_{(n)} = \left(\alpha^*\right)^{(n-1)} = \beta_{(n-1)}, \quad \alpha^* \cdot \beta^*_{(n)} = (\alpha)^{(n-1)} = \beta^*_{(n-1)}, \\ \alpha^* \cdot \beta_{(n)} = \left(\alpha^*\right)^{(n+1)} = \beta_{(n+1)}, \quad \alpha \cdot \beta^*_{(n)} = (\alpha)^{(n+1)} = \beta^*_{(n+1)}.\end{gathered} \tag{41}$$

Therefore, the unbalanced admittances for symmetric components in the positive and negative sequences, according to (39) and (40), can be presented respectively as:

$$\underline{Y}^{\mathrm{p}}_{\mathrm{u}n} = \frac{1}{3}\left[\left(\underline{Y}_{\mathrm{R}n} + \beta_{(n-1)}\underline{Y}_{\mathrm{S}n} + \beta^*_{(n-1)}\underline{Y}_{\mathrm{T}n}\right) - \underline{Y}_{en}\left(1 + \beta_{(n-1)} + \beta^*_{(n-1)}\right)\right] \tag{42}$$

$$\underline{Y}^{\mathrm{n}}_{\mathrm{u}n} = \frac{1}{3}\left[\left(\underline{Y}_{\mathrm{R}n} + \beta_{(n+1)}\underline{Y}_{\mathrm{S}n} + \beta^*_{(n+1)}\underline{Y}_{\mathrm{T}n}\right) - \underline{Y}_{en}\left(1 + \beta_{(n+1)} + \beta^*_{(n+1)}\right)\right] \tag{43}$$

The effective values of the symmetric components of current for the unbalanced component are equal:

$$\left\|\mathbf{i}^{\mathrm{p}}_{\mathrm{u}Dn}\right\| = Y^{\mathrm{p}}_{\mathrm{u}n} \cdot \left\|\mathbf{u}_{Dn}\right\|, \quad \left\|\mathbf{i}^{\mathrm{n}}_{\mathrm{u}Dn}\right\| = Y^{\mathrm{n}}_{\mathrm{u}n} \cdot \left\|\mathbf{u}_{Dn}\right\|, \quad \left\|\mathbf{i}^{\mathrm{z}}_{\mathrm{u}Dn}\right\| = Y^{\mathrm{z}}_{\mathrm{u}n} \cdot \left\|\mathbf{u}_{Dn}\right\|. \tag{44}$$

The current $\mathbf{i}_D$ flowing from D to C system has been decomposed into components describing various physical phenomena according to the relationship:

$$\mathbf{i}_D = \mathbf{i}_{\mathrm{a}D} + \mathbf{i}_{\mathrm{s}D} + \mathbf{i}_{\mathrm{r}D} + \mathbf{i}_{\mathrm{u}D}, \quad \left\|\mathbf{i}_D\right\|^2 = \left\|\mathbf{i}_{\mathrm{a}D}\right\|^2 + \left\|\mathbf{i}_{\mathrm{s}D}\right\|^2 + \left\|\mathbf{i}_{\mathrm{r}D}\right\|^2 + \left\|\mathbf{i}_{\mathrm{u}D}\right\|^2, \tag{45}$$

where $\mathbf{i}_{\mathrm{u}D} = \mathbf{i}^{\mathrm{p}}_{\mathrm{u}D} + \mathbf{i}^{\mathrm{n}}_{\mathrm{u}D} + \mathbf{i}^{\mathrm{z}}_{\mathrm{u}D}$.

These components are orthogonal, so there is a relationship between the effective values that can be presented mathematically and graphically:

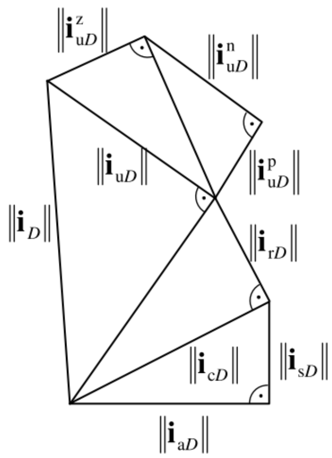

Fig. 4. Interpretation of three-phase current vectors for a four-wire connection with nonsinuso-idal periodic waveforms using multidimensional space

Taking into account N_C harmonics and equation (13) for all current components should be written:

$$\mathbf{i} = \mathbf{i}_{aD} + \mathbf{i}_{sD} + \mathbf{i}_{rD} + \mathbf{i}_{uD} + \mathbf{i}_{C}. \tag{46}$$

There is a relationship between the effective values of these currents:

$$\|\mathbf{i}\|^2 = \|\mathbf{i}_{aD}\|^2 + \|\mathbf{i}_{sD}\|^2 + \|\mathbf{i}_{rD}\|^2 + \|\mathbf{i}_{uD}\|^2 + \|\mathbf{i}_{C}\|^2. \tag{47}$$

The physical components of the current (46) are related to characteristic physical phenomena and can be dimensioned thanks to measurements at the load terminals. However, it must be emphasized that these currents do not exist physically - they are only the result of mathematical decomposition. Therefore, these are more mathematical than physical quantities. The following components were obtained:

- the vector of the active current component $\mathbf{i}_{aD}$ depends on the active power P_D of the load and, at the same time, on the resultant equivalent conductance G_e (18) of the load. The value of this conductance describes a purely resistive receiver and is directly proportional to the active power P_D and inversely proportional to the square of the effective voltage value $\mathbf{u}_D$. The effective value of this component is:

$$\|\mathbf{i}_{aD}\| = G_e\|\mathbf{u}_D\| = \frac{P_D}{\|\mathbf{u}_D\|}; \tag{48}$$

- the vector of the scattered current component $\mathbf{i}_{sD}$ (20) with an effective value equal to (21), appears when the equivalent conductance G_{en} of the load, changes with the harmonic order.

- the vector of the reactive current component $\mathbf{i}_{rD}$ (26) appears when there is a non-zero phase shift between the current $\mathbf{i}_n$ and voltage $\mathbf{u}_{Dn}$ vectors, for any harmonic in any phase. The condition for the existence of this shift is non-zero equivalent susceptance $B_{en} \neq 0$.
- the vector of the unbalanced current component $\mathbf{i}_{uD}$ appears in the case of non-zero unbalanced admittance $Y_{un} \neq 0$.
- the vector of the current component $\mathbf{i}_C$ generated by the nonlinear receiver, flowing from *C* to *D* system, determined from formula (12).
- Analogously to the current decomposition (46), decomposition in the power domain is possible, i.e. the apparent power *S* [VA] is presented by considering two directions of interaction:
- from *D* to *C* system as: active power P_D [W], reactive power Q_D [var], scattered power D_{sD} [VA] and unbalanced power D_{uD} [VA]

$$S_D = \|\mathbf{u}_D\| \cdot \|\mathbf{i}_D\| = \sqrt{P_D^2 + D_{sD}^2 + Q_D^2 + D_{uD}^2}; \tag{49}$$

- from *C* to *D* system: power S_C [VA] is returned, which acts on the internal impedance Z_D of the source (Fig. 3).

$$S_C = \|\mathbf{u}_C\| \cdot \|\mathbf{i}_C\| = \sqrt{P_C^2 + Q_C^2} \tag{50}$$

The total apparent power of the system can be presented as:

$$S^2 = \|\mathbf{u}\|^2 \|\mathbf{i}\|^2 = \left(\|\mathbf{u}_D\|^2 + \|\mathbf{u}_C\|^2\right) \cdot \left(\|\mathbf{i}_D\|^2 + \|\mathbf{i}_C\|^2\right) = \underbrace{\|\mathbf{u}_D\|^2 \|\mathbf{i}_D\|^2}_{S_D^2} + \underbrace{\|\mathbf{u}_D\|^2 \|\mathbf{i}_C\|^2 + \|\mathbf{u}_C\|^2 \|\mathbf{i}_D\|^2}_{S_{DC}^2} + \underbrace{\|\mathbf{u}_C\|^2 \|\mathbf{i}_C\|^2}_{S_C^2} \tag{51}$$

The combination of *C* and *D* systems is described by the power of mutual influence S_{DC}:

$$S_{DC} = \sqrt{\|\mathbf{u}_D\|^2 \|\mathbf{i}_C\|^2 + \|\mathbf{u}_C\|^2 \|\mathbf{i}_D\|^2}. \tag{52}$$

Ultimately, the power equation is:

$$S^2 = \underbrace{P_D^2 + D_{sD}^2 + Q_D^2 + D_{uD}^2}_{S_D^2} + S_{DC}^2 + \underbrace{P_C^2 + Q_C^2}_{S_C^2}, \tag{53}$$

where active power is:

$$P = P_D - P_C. \tag{54}$$

With this approach, the power factor is:

$$\lambda = \frac{P}{S} = \frac{P_D - P_C}{\sqrt{P_D^2 + D_{sD}^2 + Q_D^2 + D_{uD}^2 + S_{DC}^2 + P_C^2 + Q_C^2}}. \tag{55}$$

The S_C power is composed of active power P_C [W] and reactive power Q_C [var]. The P_C power acts on the R_D real part of the impedance in D system, while the Q_C power acts on the X_D imaginary.

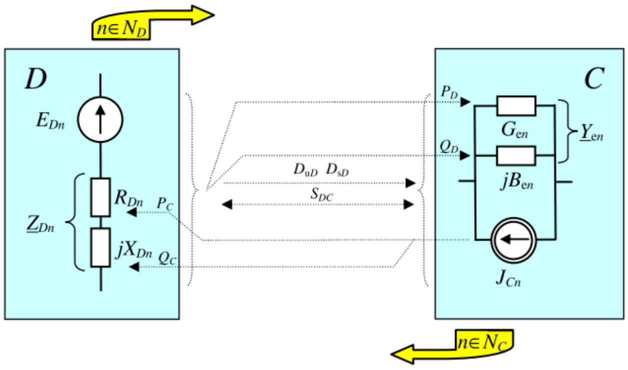

Fig. 5. Graphical representation of power behavior in a three-phase, four-wire system with periodic non-sinusoidal waveforms with a nonlinear receiver and with asymmetry

These powers have no physical interpretation. They are the result of multiplying the values of the effective physical components of current with voltage.

Both directions of energy flow should be analyzed separately. The determination of equations describing the phenomena of energy flow in particular directions must be carried out as:

- the energy flow from the nonlinear receiver to the source (from C to D system);
  The current harmonics $\mathbf{i}_C$ caused by the source J_C in each phase act on the impedance Z_D (Fig. 5). This interaction creates a potential difference, which for phase x, where x = {R, S, T}, is:

$$\underline{U}_{Cxn} = \frac{\underline{J}_{Cxn} \cdot \underline{Z}_{Dxn}}{1 + \underline{Y}_{Cxn} \cdot \underline{Z}_{Dxn}}, \quad \text{for } n \in N_C, \tag{56}$$

- energy flow from the source to the nonlinear receiver (from D to C system);
  The harmonics of the current $\mathbf{i}_D$ caused by the E_D source in each phase affect the passive Y_C. The line voltage for phase x is:

$$\underline{U}_{Dxn} = \frac{\underline{E}_{Dxn}}{1 + \underline{Y}_{Cxn} \cdot \underline{Z}_{Dxn}}, \quad \text{for } n \in N_D. \tag{57}$$

- harmonics appearing simultaneously in both E_D and J_C sources are characterized by the possibility of transmitting energy in one or the other direction, and the actual direction depends on the dominance of the source. In this case, for phase *x*, the following relationship will be:

$$\underline{U}_{xn} = \frac{\underline{E}_{Dxn} - \underline{J}_{Cxn} \cdot \underline{Z}_{Dxn}}{1 + \underline{Y}_{Cxn} \cdot \underline{Z}_{Dxn}}. \tag{58}$$

2 EXAMPLE AND DISCUSSION

A single-phase nonlinear load was connected to a real three-phase source (Fig. 6), whose internal impedance $\underline{Z}_D$ consists of resistance $R_D = 0.01\ \Omega$ and inductance $L_D = 0.5/\pi$ mH. The pulsation of the source is $\omega = 2\pi 50$ rad/s, while a symmetrical three-phase voltage source is:

$$e_\mathrm{R} = \sqrt{2}\left[230\sin(\omega t) + 80\sin(2\omega t) + 30\sin(5\omega t)\right]\mathrm{V},$$
$$e_\mathrm{S} = \sqrt{2}\left[230\sin\left(\omega t + \frac{2\pi}{3}\right) + 80\sin\left(2\omega t + 2\frac{2\pi}{3}\right) + 30\sin\left(5\omega t + 5\frac{2\pi}{3}\right)\right]\mathrm{V},$$
$$e_\mathrm{T} = \sqrt{2}\left[230\sin\left(\omega t + \frac{4\pi}{3}\right) + 80\sin\left(2\omega t + 2\frac{4\pi}{3}\right) + 30\sin\left(5\omega t + 5\frac{4\pi}{3}\right)\right]\mathrm{V}.$$

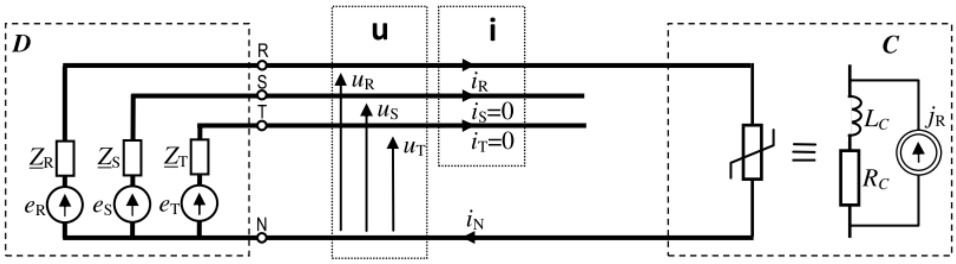

Fig. 6. The nonlinear single-phase receiver connected to a three-phase power source

The nonlinear receiver was powered from the R phase. It has resistance $R_C = 0.1\ \Omega$ and inductance $L_C = 0.001/\pi$ H, and generates harmonic current into the power grid:

$$j_\mathrm{R} = \sqrt{2}\left[10\sin(2\omega t) + 2\sin(7\omega t)\right]\mathrm{A}.$$

The impedances and admittances of *C* and *D* systems are:

n	1	2	5	7
$\underline{Z}_{Dn}$ [Ω]	0.01+*j*0.05	0.01+*j*0.1	0.01+*j*0.25	0.01+*j*0.35
$\underline{Y}_{Cn}$ [S]	5–*j*5	2–*j*4	0.3846–*j*1.9231	0.2–*j*1.4

The first and fifth harmonics appear only in the D system, so equality (57) will be used to determine the $\underline{U}_R$ voltage value, for the seventh harmonic - equality (56), and for the second harmonic - equality (58). The voltages in the remaining phases are equal to the voltages of the $\underline{E}_D$ sources.

n	1	2	5	7
$\underline{U}_R$ [V]	172.83–j26.59	55.48–j6.96	20.15–j1.04	0.03–j0.47
$\underline{U}_S$ [V]	–115+j199.19	–40–j69.28	–15–j25.98	0
$\underline{U}_T$ [V]	–115–j199.19	–40+j69.28	–15+j25.98	0
$\underline{I}_R$ [A]	731.21–j997.11	73.14–j235.85	5.74–j39.16	–1.34+j0.05
P_R [W]	$152.89\cdot 10^3$	5698.84	156.63	–0.02

The current in the R phase was determined from the relationship $\underline{I}_{Rn} = \underline{U}_{Rn}\underline{Y}_{Cn} - J_{Rn}$, and the active power from the relationship $P_{Rn} = \mathrm{Re}\{\underline{U}_{Rn}\underline{I}^{*}_{Rn}\}$.

The second harmonic current is generated by the D system and is partially reduced by the nonlinear load by the value $\underline{J}_{R,2} = 2$ A. However, this does not change the direction of energy flow because $P_{R,2} > 0$. Changing the direction of energy flow is occurs for the seventh harmonic $n = 7$.

The active powers are equal to the powers in the R phase and according to formulas (11) and (12) are:

$P_D = P_{R,1} + P_{R,2} + P_{R,5} = 158745.65$ W,

$P_C = -P_{R,7} = 17.94$ mW,

$P = P_D - P_C = 158745.63$ W.

As you know, active power is the power responsible for the effective part of electrical energy that the load actively uses for energy transformation. However, not every receiver is able to use energy with higher harmonics. For example, a synchronous or asynchronous machine actively uses only the power P_1 of the fundamental harmonic. The remaining harmonics have an unfavorable effect, causing disturbances in the rotating magnetic flux, which translates into deterioration of efficiency. In this example, the active power that is not actively used in the receiver is:

$$P_{\mathrm{H}} = \sum_{n\neq 1} |P_n| = 5855.49 \text{ W},$$

which is 3.7% of the total power P. Assuming a proportional relationship between the engine torque and its power, this means a 3.7% drop in torque, even though the active energy consumption counter will count the total power P_1+P_{H}.

For a single-phase load according to equation (23), the equivalent admittance $\underline{Y}_{en}$ is equal to the admittance $\underline{Y}_{Cn}$ in the R phase.

The effective value of voltage $\mathbf{u}_D$ is determined from (7) and (11). For this purpose, only those voltage components that cause energy to flow from D to C system should be used. The flow in this direction takes place only in the R phase for the 1st, 2nd and 5th harmonics.

$$\|\mathbf{u}_D\| = \sqrt{\sum_{n\in N_D} \|\mathbf{u}_n\|^2} = \sqrt{\sum_{n\in N_D} \left(\|\mathbf{u}_{\mathrm{R}n}\|^2 + \|\mathbf{u}_{\mathrm{S}n}\|^2 + \|\mathbf{u}_{\mathrm{T}n}\|^2\right)} = \sqrt{\sum_{n=1,2,5} \|\mathbf{u}_{\mathrm{R}n}\|^2} = 184.69 \text{ V}.$$

From (19), the value of the equivalent conductance is $G_e = 4.65$ S, while from (48) the active power P_D forces the flow of current $\mathbf{i}_{aD}$ with an effective value equal 859.5 A.

The effective value of the scattered component $\mathbf{i}_{sD}$ according to (21) is equal 181.96 A, while according to (26) the effective value of the reactive current component $\mathbf{i}_{rD}$ is equal 903.32 A. The current $\mathbf{i}_D$ is determined from (11) and is equal 1261.52 A. This means that the unbalanced component is equal

$$\|\mathbf{i}_{uD}\| = \sqrt{\|\mathbf{i}_D\|^2 - \|\mathbf{i}_{aD}\|^2 - \|\mathbf{i}_{sD}\|^2 - \|\mathbf{i}_{rD}\|^2} = 60.01 \text{ A}.$$

The component of the current flowing in the opposite direction is determined for the 7th harmonic and is

$$\|\mathbf{i}_C\| = |\underline{I}_{R,7}| = 1.34 \text{ A}.$$

The individual power components are:

$$S = \|\mathbf{u}\| \cdot \|\mathbf{i}\| = \sqrt{\|\mathbf{u}_D\|^2 + \|\mathbf{u}_C\|^2} \cdot \sqrt{\|\mathbf{i}_D\|^2 + \|\mathbf{i}_C\|^2} = 232997.56 \text{ VA},$$
$$S_D = \|\mathbf{u}_D\| \cdot \|\mathbf{i}_D\| = 232996.68 \text{ VA},$$
$$P_D = 158745.65 \text{ W},$$
$$D_{sD} = \|\mathbf{u}_D\| \cdot \|\mathbf{i}_{sD}\| = 33606.65 \text{ VA},$$
$$Q_D = \|\mathbf{u}_D\| \cdot \|\mathbf{i}_{rD}\| = 166838.31 \text{ var},$$
$$D_{uD} = \|\mathbf{u}_D\| \cdot \|\mathbf{i}_{uD}\| = 11083.47 \text{ VA},$$
$$S_C = \|\mathbf{u}_C\| \cdot \|\mathbf{i}_C\| = 628.28 \text{ mVA},$$
$$P_C = -\Re e\{\underline{U}_{R,7} \cdot \underline{I}^*_{R,7}\} = 17.94 \text{ mW},$$
$$Q_C = -\Im m\{\underline{U}_{R,7} \cdot \underline{I}^*_{R,7}\} = 628.03 \text{ mvar},$$
$$S_{DC} = \sqrt{\|\mathbf{u}_D\|^2 \|\mathbf{i}_C\|^2 + \|\mathbf{u}_C\|^2 \|\mathbf{i}_D\|^2} = 641.33 \text{ VA},$$
$$P = 158745.63 \text{ W}.$$

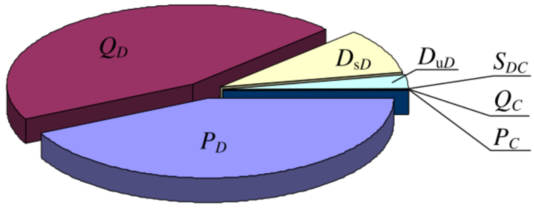

Fig. 7. Decomposition of power components in the power balance

According to (55), the power factor is $\lambda = P/S = 0.681$.

Classic power compensation [10], [12], which involves zeroing the Q_D power, improves the power factor to 0.976. Performing two-stage compensation according to [16] resets the Q_D and D_{uD} power to zero, which improves the power factor to 0.978. The use of a series compensator as another additional element [18] makes it possible to additionally reset the D_{sD} power to zero, which results in an improvement of the power factor to the value 0.9999917. Further improvement of the power factor is impossible.

3 CONCLUSION

The method of analysis adopted, based on knowledge of the parameters of source J_C, may be questionable. Determining these parameters in reality may be difficult. The J_C source parameters actually depend on the values of other elements in the circuit and have an implicit functional dependence on: parameters of *D* system, power grid impedance, impedance of *C* system and other harmonics. For this reason, the parameters of the J_C source are not stationary and change their values as a result of changes in any parameter in the circuit.

The ambiguity in the description of the electric circuit in various power theories results from the lack of a comprehensive mathematical description of the power system. Therefore, research in this field is still justified and needed.

SYMBOLS

$\mathbf{1}^{\mathrm{p}},\mathbf{1}^{\mathrm{n}},\mathbf{1}^{\mathrm{z}}$	three-phase unite symmetrical vectors
α	120° rotation operator
B	susceptance, S
β	generalized complex rotation coefficient
D_{u}	unbalanced power, VA
D_s	scattered power, VA
e	instantaneous value of the voltage source, V
E	effective value of the voltage source, V
G	conductance, S
i	instantaneous value of the line currents, A
$\mathbf{i}$	vector of instantaneous currents in a three-phase system, A
$\mathbf{i}_{\mathrm{a}}$	active component of current - three-phase vector, A
$\mathbf{i}_{\mathrm{s}}$	scattered component of current - three-phase vector, A
$\mathbf{i}_{\mathrm{r}}$	reactive component of current - three-phase vector, A
$\mathbf{i}_{\mathrm{u}}$	unbalanced component of current - three-phase vector, A
I	effective value of phase current, A
$\underline{\mathbf{I}}$	vector of complex currents in a three-phase system, A
j	instantaneous value of the current source, A
J	effective value of the current source, A
L	inductance, H
m	multiple of active phases
P	active power, W
Q	reactive power, var

R	resistance, Ω
S	three-phase apparent power, VA
u	instantaneous voltage values relative to the virtual star point, V
$\mathbf{u}$	vector of instantaneous voltages in a three-phase system, V
U	effective value of phase voltage, V
$\underline{\mathbf{U}}$	vector of complex voltages in a three-phase system, V
Y	admittance, S
Z	impedance, Ω

Subscripts

R,S,T,N	phase and neutral wires
D	a set of components describing the flow of energy from D system to C system
C	a set of components describing the flow of energy from C system to D system

Acronyms

THD	Total Harmonic Distortion
CPC	Currents Physical Components

Bibliography

[1] Fryze S. (1931) „Moc rzeczywista, urojona i pozorna w obwodach elektrycznych o przebiegach odkształconych prądu i napięcia”. Przegląd Elektrotechniczny 7
[2] Buchholz F. (1950) “Das Begriffsystem Rechtleistung, Wirkleistung, totale Blindleistung”. Selbstverlag, Munchen, Germany
[3] Czarnecki L.S. (1989) “Reactive and unbalanced currents compensation in three-phase circuits under nonsinusoidal conditions”. IEEE Trans. IM, IM-38, No. 3, pp. 754-459.
[4] Czarnecki L.S. (1999) “Energy flow and power phenomena in electrical circuits illusions and reality”, Archiv fur Elektrotechnik 82(4)
[5] IEEE Standard Definitions for the Measurement of Electric Power Quantities under Sinusoidal Non-sinusoidal, Balanced or Unbalanced Conditions (2002), IEEE Std. 1459-2000
[6] Czarnecki L.S. (2005) „Moce w obwodach elektrycznych z niesinusoidalnymi przebiegami prądów i napięć”. Oficyna Wydawnicza Politechniki Warszawskiej
[7] Balci M.E., Hocaoglu M.H. (2009) “Comparative review of multi-phase apparent power definitions”. International Conference on Electrical and Electronics Engineering - ELECO 2009, pp. 144-148, doi: 10.1109/ELECO.2009.5355309.
[8] Emanuel A.E. (2010) “Power definitions and the physical mechanism of power flow”. IEEE Press, John Wiley & Sons, Ltd.
[9] Czarnecki L.S., Toups T.N. (2014) “Working and reflected active powers of harmonics generating single-phase loads”. Przegląd Elektrotechniczny 90(11)
[10] Zajkowski K. (2017) “Settlement of reactive power compensation in the light of white certificates”. E3S Web of Conferences, 19, doi: 10.1051/e3sconf/20171901037

[11] Czarnecki L.S. (2018) „Współczynnik mocy odbiorników elektrycznych”. Automatyka, Elektryka, Zakłócenia. Bezpieczeństwo, pomiary i niezawodność w elektroenergetyce., Gdańsk, Poland, pp. 38-47

[12] Zajkowski K., (2018) “Reactive power compensation in a three-phase power supply system in an electric vehicle charging station”. The Journal of Mechanical and Energy Engineering, 2(42), pp. 75–84. doi: 10.30464/jmee.2018.2.1.75

[13] Zajkowski K., et al. (2018) “The use of CPC theory for energy description of two nonlinear receivers”. MATEC Web of Conferences. doi: 10.1051/matecconf/201817809008

[14] Zajkowski K., Rusica I. (2019) “The method of calculating LC parameters of balancing compensators in a three-phase four-wire circuit for an unbalanced linear receiver”. IOP Conference Series: Materials Science and Engineering, 564, 12134. doi: 10.1088/1757-899x/564/1/012134

[15] Zajkowski K., Rusica I. (2019) “Comparison of electric powers measured with digital devices relative topowers associated with distinctive physical phenomena”. IOP Conference Series: Materials Science and Engineering, pp. 1–6. doi: 10.1088/1757-899x/564/1/012133

[16] Zajkowski K. (2020) “Two-stage reactive compensation in a three-phase four-wire systems at nonsinusoidal periodic waveforms”. Electric Power Systems Research, 184: 1–9. doi: 10.1016/j.epsr.2020.106296

[17] Czarnecki L.S., Almousa M. (2021) “Adaptive Balancing by Reactive Compensators of Three-Phase Linear Loads Supplied by Nonsinusoidal Voltage from Four-Wire Lines”. American Journal of Electrical Power and Energy Systems, 10(3), pp. 32-42

[18] Zajkowski K., Duer S. (2023) “Decomposition of the Voltages in a Three-Phase Asymmetrical Circuit with a Non-Sinusoidal Voltage Source”. Energies 16(22), 7616, doi: 10.3390/en16227616

[19] Zajkowski K., et al. (2023) “Cooperation of a Non-Linear Receiver with a Three-Phase Power Grid”, Energies 16(3), 1418, doi: 10.3390/en16031418

[20] Zajkowski K. (2014) “The method of solution of equations with coefficients that contain measurement errors, using artificial neural network”. Neural Computing and Applications, 24(2), pp. 431-439, doi: 10.1007/s00521-012-1239-0

[21] Smyczek J., Zajkowski K. (2004) “Simulation of overvoltages for switching off lagging load from mains”. 2nd International Industrial Simulation Conference, pp. 278–281

[22] Hanzelka, Z., Milanović, J. (2008) Principles of Electrical Power Control. In: Strzelecki, R., Benysek, G. (eds) Power Electronics in Smart Electrical Energy Networks. Power Systems. Springer, London, doi: 10.1007/978-1-84800-318-7_2

[23] Duer S., et al. (2013) “Designing of an effective structure of system for the maintenance of a technical object with the using information from an artificial neural network”. Neural Computing and Applications, 23(3-4), pp. 913-925, doi: 10.1007/s00521-012-1016-0

[24] Duer S., Zajkowski K. (2013) “Taking decisions in the expert intelligent system to support maintenance of a technical object on the basis information from an artificial neural network”. Neural Computing and Applications, 23(7-8), pp. 2185-2197, doi: 10.1007/s00521-012-1169-x

[25] Siergiejczyk M., et al. (2015) “Modeling of Process of Maintenance of Transport Systems Telematics with Regard to Electromagnetic Interferences, In: Mikulski, J. (eds) Tools of Transport Telematics”. Communications in Computer and Information Science, 531, pp. 99-107, doi: 10.1007/978-3-319-24577-5_10
[26] Siergiejczyk M., et al. (2016) “Issue of reliability–exploitation evaluation of electronic transport systems used in the railway environment with consideration of electromagnetic interference”. IET Intell. Transp. Syst., 10, pp. 587–593, doi: 10.1049/iet-its.2015.0183
[27] Zajkowski K. (2017) “An innovative hybrid insulation switch to enable/disable electrical loads without overvoltages”. E3S Web of Conferences, 19. doi: 10.1051/e3sconf/20171901033
[28] Duer S., et al. (2018) “Analyses of the method development of decisions in an expert system with the use of information from an artificial neural network”. MATEC Web of Conferences, doi: 10.1051/matecconf/201817807002
[29] Krzykowski M., et al. (2019) “Assessment of the level of reliability of power supplies of the objects of critical infrastructure”. IOP Conference Series: Earth Environ. 214, 012018, doi: 10.1088/1755-1315/214/1/012018
[30] Montoya F.G., et al. (2021) “Vector Geometric Algebra in Power Systems: An Updated Formulation of Apparent Power under Non-Sinusoidal Conditions”. Mathematics 9, 1295. doi: 10.3390/math9111295
[31] Soljan Z., et al. (2021) “CPC-Based Minimizing of Balancing Compensators in Four-Wire Nonsinusoidal Asymmetrical Systems”. Energies 14(7), 1815, doi: 10.3390/en14071815
[32] Stawowy M., et al. (2021) “Method of Estimating Uncertainty as a Way to Evaluate Continuity Quality of Power Supply in Hospital Devices”. Energies 14(2), 486, doi: 10.3390/en14020486
[33] Duer S., et al. (2021) “Examination of Multivalent Diagnoses Developed by a Diagnostic Program with an Artificial Neural Network for Devices in the Electric Hybrid Power Supply System “House on Water”. Energies 14, doi: 10.3390/en14082153
[34] Duer S., et al. (2022) “Organization and Reliability Testing of a Wind Farm Device in Its Operational Process”. Energies 15, doi: 10.3390/en15176255
[35] Duer S., et al. (2022) “Intelligent Systems Supporting the Use of Energy Devices and Other Complex Technical Objects: Modeling, Testing, and Analysis of Their Reliability in the Operating Process”. Energies 15, doi: 10.3390/en15176414
[36] Gubarevych O., et al. (2023) “Research and assessment of the reliability of railway transport systems with induction motors”. Energies 16(19), 6888, doi: 10.3390/en16196888
[37] Duer S., et al. (2023) “Reliability Testing of Wind Farm Devices Based on the Mean Time to Failures”. Energies 16(6), 2827, doi: 10.3390/en16062827
[38] Duer S., et al. (2023) “Assessment of the Reliability of Wind Farm Device on the Basis of Modeling Its Operation Process”. Energies 16(1), 142, doi: 10.3390/en16010142
[39] Siergiejczyk M., et al. (2014) “Evaluation of Safety of Highway CCTV System's Maintenance Process”. Telematics - Support For Transport, 471, pp. 69-79
[40] Duer S., et al. (2023) “Reliability Testing of Wind Farm Devices Based on the Mean-time Between Failures (MTBF)”. Energies 16(4), 1659, doi: 10.3390/en16041659