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MODELLING AND SIMULATION OF ADHESION OF THE RAIL VEHICLE

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Abstract – In the paper we consider the issue related to the phenomenon of wheel-to-rail adhesion. This issue concerns self-excited vibrations in the area of wheel-rail contact, caused by unstable friction. Such friction is characterized by decreasing values of the friction coefficient with increasing slipping velocity. The paper includes mathematical formulation of the problem and presents the method for solving it. In addition, a simulation of elementary vehicle driving was performed. During the simulation, self-excited vibrations arose during wheel slip caused by high driving torque. The paper presents graphs illustrating the tangential force and the slipping velocity during these vibrations.

Keywords – modelling, self-excited vibrations, simulation, wheel adhesion, unstable friction

INTRODUCTION

This work discusses a model of a tangential longitudinal interaction between the vehicle wheel and the rail. The character of this phenomenon known as the "grip" is determined by the friction between the wheel and the rail and their local deformations, caused by the propelling or braking forces acting on the vehicle.

Grip is characterized by the function determining the relationship between the tangential force and the velocity of the wheel slip. Complexity of the problem of grip is the reason why only the basic characteristic determining the relationship between the longitudinal force and the wheel slip speed is used to analyze the driving and braking processes.

$$v_s = r\omega - v, \tag{1}$$

where:

r – wheel radius,
ω – angular velocity of the wheel,
v – velocity of the wheel center.

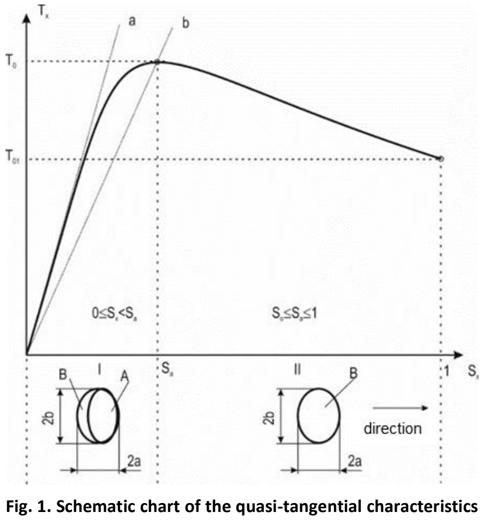

Fig. 1. Schematic chart of the quasi-tangential characteristics of grip and of the elliptic trail of adhesion ; A – adhesion patch (no slip), B – slip patch; I, II designations of the slip intervals (authors' data)

A quasi-static characteristic of grip is shown in the form of a set of function graphs, whose argument is a relative slip. These functions are determined under various fixed conditions of the wheel motion and pressure.

$$s := \frac{v_s}{v_0}, \qquad (2)$$

where:

$v_0 := r\omega$, when the wheel is driven,

$v_0 := v$ when brakes are applied to the wheel.

Characteristics are determined on the basis of the measurement results or based on the computation results according to the Kalker theory [7, 10, 11, 14]. A schematic diagram of the discussed characteristic is shown in Figure 1 [2].

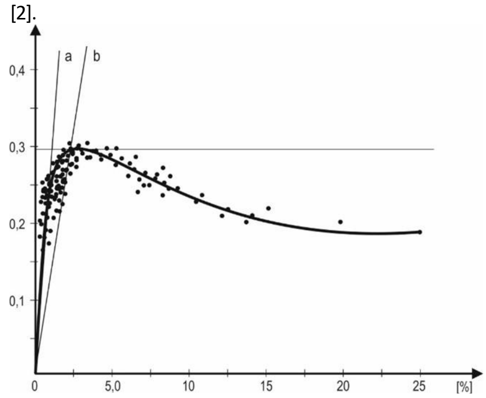

Fig. 2. Measurement data of μ coefficient [11] during motion with velocity v=36 km/h (authors' data)

The essential parameters, determining the plot, have been marked in the chart, which represents the driven wheel. The greatest value of the grip force T_o is generated when $s=s_o$. The value of quotient $\mu_o:=T_o/Q$ determines the most important of the grip characteristics, which is a coefficient of adhesive friction between the wheel and the rail. The second boundary value amounting to $T_{o1}<T_o$, determines the grip force where there is a full wheel slip, i.e. when s=1 (or v=0, rω>0). Quotient $\mu_1:=T_{o1}/Q$ determines the value of the sliding friction coefficient. The chart shown in Figure 1 has been divided into two sections: - ($0\le s<s_o$) in the first interval, the wheel rolls with slip, which is mainly due to wheel and rail deformations, and on the trail of adhesion, there are patches: the adhesion and slip patches, - in the second interval ($s_0<s\le 1$), on the adhesion trail there is only the slip present. Next to the chart, there are two straight lines marked: a, b. The straight line a is a tangent to the chart at (0,0), while the straight line b is a chord through the point (s_o, T_o). The chart in Figure 1 is determined on the basis of computations, whereas the chart in Figure 2 is determined based on the results of the experimental research. Usually, on the vertical axes in these charts, there are, instead of forces T, the values of quotient μ:=T/Q given [13].

Figure 2 shows the results of measurements of μ coefficient, presented in [11]. It illustrates the points where measurements were taken and the line approximating the measurement results. Very often, the Magic Formula [9] is used to determine the description of the approximating function. The form of this chart is analogous to the chart in Figure 1. It allows for determination of values of the parameters discussed earlier in this paper, e.g. $\mu_o\approx0{,}3$, $s_o\approx0{,}025$ as well as angle of inclination of the tangent a.

I. DYNAMIC MODEL OF GRIP

As mentioned earlier in this work, the model of grip discussed in the section above, is of the quasi-static character, which means, that it is applicable under the fixed conditions of the wheel motion and load. Such a form of the grip characteristic does not comprise dynamic effects of grip, resulting from the friction between the wheel and rail, nor their deformability. Besides, it needs to be observed that this characteristic cannot be directly used in simulation tests of the car take off, when $v_s=0$. Apart from this, the quasi-static model does not address the dynamic phenomena caused by the sliding friction with the friction coefficient decreasing as the speed of the wheel sliding on the rail increases. In the further discussion of the subject, the authors will propose the grip model enabling analysis of these phenomena, the self-excited vibrations in particular, that may occur during the wheel slip. This grip model will be referred to as dynamic. The mathematical description of the considered model of grip will be presented using the relationships between the longitudinal tangential force T, the resultant wheel slip velocity v_s, which can be viewed as the slip caused by elastic deformations of the wheel and rail (creep) [12] v_{se}, and as the frictional wheel slip v_{ss}, that is

$$v_s = v_{se} + v_{ss}, \qquad (3)$$

Relationships between force T and the types of slip mentioned above are adopted on the basis of [3, 4, 6] in the form

$$v_{se} = \alpha(r\omega + v)T + \beta\dot{T}, \qquad (4a)$$

$$T \in \Gamma(v_{ss}). \qquad (4b)$$

where:

α, β – constants characterizing the elastic features of the wheel and rail,

$T \in \Gamma(v_{ss})$ representation determining the features of the wheel-rail couple, determined by the charts in Figure 3 or formulae 5a and 5b.

a)

b)

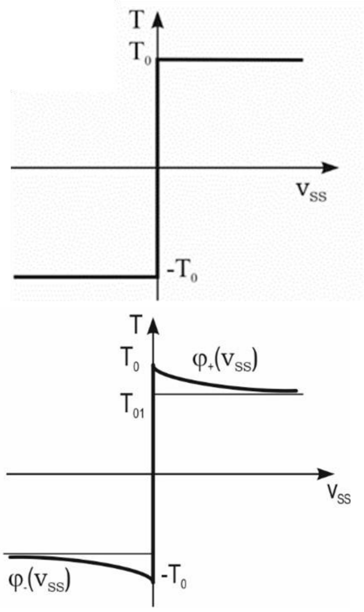

Fig. 3. Charts of relationships characterizing the wheel and rail friction a) according to Coulomb, b) unstable friction (authors' data)

$$\Gamma(v_{ss}) := \begin{cases} [-T_0, +T_0] & - \text{ for } v_{ss} = 0 \\ \{\varphi(v_{ss})\} & - \text{ for } v_{ss} \neq 0 \end{cases}, \tag{5a}$$

Usually function ϕ is adopted in the form

$$\varphi(v) := \left[T_0 - (T_0 - T_{01})\left(1 - e^{-|v|/v_{0s}}\right)\right]\operatorname{sign}(v) \tag{5b}$$

In the formula (5b), the following parameters describing friction occur:

Q – wheel pressure on the rail,

μ_0, μ_{01} - boundary values of the friction coefficient,

v_{os} - constant characterizing the exponential curve.

Based on (3) and (4) an equation can be formulated determining relationships between the longitudinal force T and the wheel slips on the rail.

$$\beta\dot{T} + \alpha(r\omega + v)T + v_{ss} = r\omega - v, \tag{6a}$$

$$T \in \Gamma(v_{ss}). \tag{6b}$$

The foundation for designing the above model of the wheel and rail grip was the works [3, 4, 6] as well as [5], where the analogous description of the grip of a wheel with a tyre and a road was analyzed and verified.

II. DESCRIPTION OF THE ELEMENTARY VEHICLE'S MOTION

Here, the elementary vehicle with one driving axle, on which the given driving or braking torque acts, is being considered.

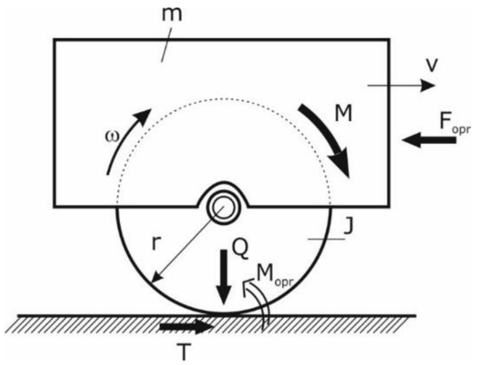

Fig. 4. Elementary vehicle (authors' data)

Figure 4 shows such a model of vehicle, which is determined by the following quantities: m – vehicle mass reduced to one drive axle, J – moment of inertia reduced to the axles of the wheels in the set, and the moment of inertia of the drive system components, M – drive torque reduced to the wheel axle, ω – angular velocity of the wheel, v – vehicle velocity, T – tangential force acting on the wheel, F_{opr} – force of the motion resistance, M_{opr} - rolling resistance torque of the wheel set.

The form of the motion description of the elementary vehicle is relatively uncomplicated and comprises motion equations as well as the grip description. In the model of the elementary vehicle, it has been assumed, that driving forces affecting the left and the right wheel are equal.

$$m\dot{v} = 2T - F_{opr}, \tag{7a}$$

$$J\dot{\omega} = M - 2Tr - M_{opr}, \tag{7b}$$

$$\beta\dot{T} + \alpha(r\omega + v)T + v_{ss} = r\omega - v, \tag{7c}$$

$$T \in \Gamma(v_{ss}). \tag{7d}$$

On the basis of these equations for a given function M defining the time waveform of the drive torque, a solution determining waveforms of velocity v, ω and force T, and also variable v_{ss} affecting the friction slip of the wheel, can be established.

The above-mentioned specifics of the relationship (4b) ensues, that variables T and v_{ss} change abruptly, that is, they are discontinuous in certain states determined by the points of the friction characteristics (Figure 3b).

These discontinuities remain when $|T|=T_o$ and $T\dot{T}>0$, which directly results from the waveform shown in Figure 3b.

III. SIMULATION OF THE VEHICLE MOTION

Description of the vehicle motion depicted in formulae (7) will be presented in the form adequate for numerical computations. Due to the above-mentioned discontinuities of function T and v_{ss}, related to friction defined by the chart (Figure 3b) it is more convenient considering equations 7c and 7d in two phases. The first phase is determined by relationships $v_{ss}=0$, $|T|<T_o$, when the wheel rolls with elastic slip (pseudo slip), and equation 7c has the form

$$\beta\dot{T} + \alpha(r\omega + v)T = r\omega - v \quad \text{for} \quad vss = 0, |T| < To, \tag{8}$$

The second phase is determined by conditions $|v_{ss}|>0$, $T_{o1}<|T|<T_o$, the wheel rolls with the pseudo slip and the friction slip, and equation (7c) has now the form

$$\beta\dot{T} + \alpha(r\omega + v)T + \varphi^{-1}(T) = r\omega - v. \tag{9}$$

where:

ϕ^{-1} – inverse function relative to function ϕ defined in formula (5).

To conduct the simulation experiment, the following parameter values were adopted, corresponding to the basic technical data of the EMU (electric multiple unit) consisting of four modules with the Bo'2'2'2'Bo' axle system (two extreme driving bogies, the other three rolling bogies), with the mass m_p=172 t, four 500 kW motors, the gear with ratio z_p=5,8, radius of the driving wheel r_k=0,420 m. On the basis of the above it has been established that m=43000 kg – vehicle mass reduced to one drive axle, Q=84366 N – wheel set pressure on one wheel, T_0=22000 N, T_{01}=13200 N – boundary values of the frictional force, α=0,000001·1/N, β=0,0000007 m/N – elastic constants of the wheel and spring, J=1210 kgm^2, - moment of inertia (engine rotor, transmission, drive wheels with axle, F_{opr}=0 - rolling resistance has been omitted.

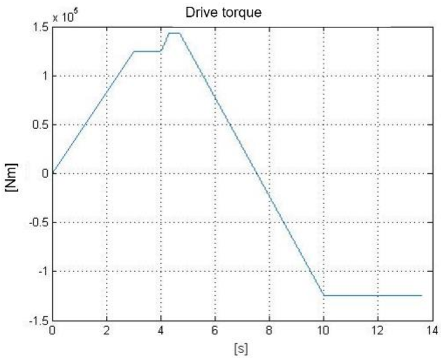

Fig. 5. Reduced drive torque M (authors' data)

Figure 5 shows the chart of the engine torque with values reduced to the one wheel $F_M= z_p M_s/(2 r)$.

The solution of equations (7) is presented in the force chart in Figure 6, in the chart of the angular velocity of the engine ω and the vehicle velocity (Figure 7) for the full driving distance lasting 13.7 s.

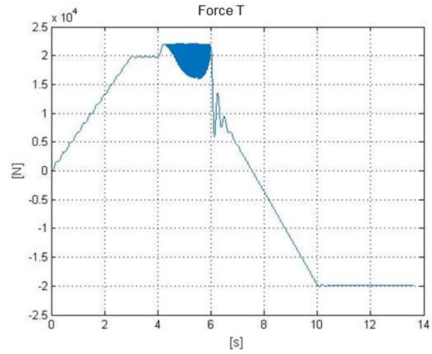

Fig. 6. The frictional force T affecting the wheel, self-excited vibrations occur in interval [t_1, t_2] (authors' data)

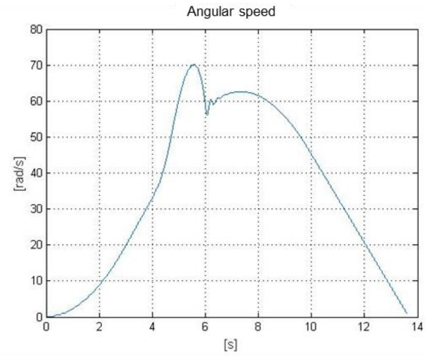

Fig. 7. Charts of the engine angular speed ω and the vehicle velocity v (authors' data)

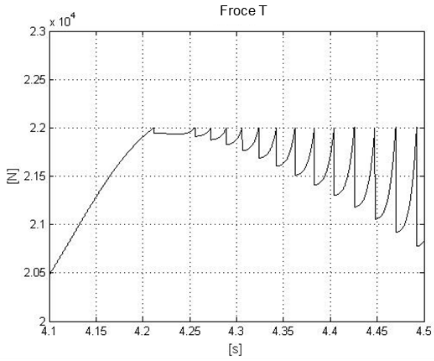

Fig. 8. Magnified chart of frictional force T, slip velocity v_{ss} during self-excited vibrations in the interval [t_1, t_2], identified in Figure 6 (authors' data)

Figure 8, however, shows the waveform of the force T and the velocity of slip v_{ss} in the interval between 4.1 s to 4.5 s. Up to the t_1=4,25 s (Figure 8) engine driving torque produces force T, whose value does not exceed the boundary value T_0=22000 N. In this time span the slip velocity v_{ss}=0. After t_1, the engine torque still increases, whereas the value of force T and of the slip velocity v_{ss} change abruptly, because the self-excited vibrations are generated due to friction with the characteristics shown in Figure 3b.

The process of self-excited vibration ends at t_2=6 s, when the driving torque M diminishes - Figure 6.

After t_2, the value of engine torque does not exceed the boundary value, as a result of which self-excited vibrations do not occur and a slip velocity v_{ss} is equal to zero. The presented charts illustrate the dynamic effects to be observed between the wheel and the rail in the course of the wheel slippage.

IV. Conclusions

The results of the calculations presented in this work show that self-excited vibrations arise when the values of the driving or braking torque exceed the boundary value $M_0=2\ T_0\ r/z_p$.

Situations of this kind may occur quite frequently when the vehicle is operated. They are related to random factors which can affect the change in friction between the wheel and the rail, which causes additional loads to the vehicle's drive system [8]. The discussed model of dynamic grip enables the simulation research of additional loads affecting the drive system caused by self-excited vibrations. Vibration of this kind occur during the wheel slip in the course of accelerating or applying the brakes on the rail vehicle.

Bibliography

[1] Carter F. W., On the action of locomotive driving wheel, Proceedings of Royal Society of London, London 1926.

[2] Garg V. K., Dukkipati R. V., Dynamics of railway vehicle systems, Academic Press 1984.

[3] Grzesikiewicz W., Hamulce pojazdów szynowych, Wydawnictwo Politechniki Warszawskiej 1982.

[4] Grzesikiewicz W., Piotrowski J., On the Development of the traction Force between Wheel and Substrate, A Selection of Simple Mathematical Models. Machine Dynamics Problems, vol.17 1997.

[5] Grzesikiewicz W., Modelowanie stycznego oddziaływania koła na podłoże w ruchu nieustalonym, Konstrukcja, badania eksploatacja, technologia pojazdów, Teka Komisji Naukowo-Problemowej Motoryzacji, Zeszyt nr 18, Kraków 1999.

[6] Grzesikiewicz W., Wakulicz A., Modelowanie samowzbudnych drgań koła pojazdu, Zeszyt Nauk Katedry Mechaniki Stosowanej, Nr 23/2004 Politechnika Śląska, Gliwice 2004.

[7] Kalker J. J., Three-dimensional elastic Bodies in Rolling Contact, Kluwer Academic Publisher Dordrecht 1990.

[8] Lewandowski M., Analiza zjawisk elektromechanicznych w szynowym pojeździe trakcyjnym z uwzględnieniem zmian współczynnika przyczepności kół napędowych., z. 139 OWPW, Warszawa 2009.

[9] Pacejka H. B., Tyre and Vehicle Dynamics, Butterworth-Heiman 2002.

[10] Piotrowski J., Kalker's algorithm Fastsim tangential contact problems with slip-dependent friction and friction anisotropy, Vehicle Systems Dynamics Vol. 48 No 2010.

[11] Polach O., Creep forces in simulations of traction vehicle running on adhesion limit, WEAR, Elsevier 258, 2005.

[12] Rocard Y., La stabilite´ de route des Locomotives, Herman, Paris 1935.

[13] Rocard Y., L´instabilite´ en macanique, Masson, Paris 1954.

[14] Wickens A. H., Fundamentals of rail Vehicle Dynamics, Swets & Zeitlinger 2013.