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NEW STABILITY CONDITIONS FOR THE DISCRETE-TIME AND CONTINUOUS-TIME LINEAR ROESSER MODELS

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Abstract – New simple sufficient conditions of the asymptotic stability of the discrete-time and continuous-time linear 2D Roesser models are proposed. The effectiveness of the new conditions are demonstrated on simple numerical examples.

Key words – asymptotic stability, sufficient conditions, the stability of 2-D Roesser models.

1. INTRODUCTION

The most popular models of linear two-dimensional (2-D) systems are the models introducted by Roesser in 1975[14]. Fornasini and Marchasini models in 1978 [2] and Kurek in 1985 [12]. An overview of 2D linear systems theory is given in [3,5-7,15-17]. The switch linear systems described by the Roesser models have been analyzed in [4,5]and positive descriptor systems in [16].The fractional systems have been analyzed in [7,15,16] and positive switch 2D linear system in [7].

In this paper new simple sufficient conditions of the asymptotic stability of the discrete-time and continuous-time have been proposed. The paper is organized as follows. In Section 2 basic definitions and theorems concerning discrete-time and continuous-time linear 2D Roesser models are recalled. New sufficient conditions of the asymptotic stability of the discrete-time 2D Roesser model are given in Section 3 and of the continuous-time 2D Roesser model in Section 4. Concluding remarks are given in Section 5.The following notation will be used: $\Re$ - the set of real numbers, $\Re^{n\times m}$ - the set of $n\times m$ real matrices, I_n - the $n\times n$ identity matrix.

2. THE DISCRETE-TIME ROESSER MODEL

Consider the discrete-time 2D Roesser model[14]

$$\begin{bmatrix} x_{i+1,j}^h \\ x_{i,j+1}^v \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} x_{ij}^h \\ x_{ij}^v \end{bmatrix} + \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} u_{ij}, \tag{2.1a}$$

$$y_{ij} = [C_1 \quad C_2]\begin{bmatrix} x_{ij}^h \\ x_{ij}^v \end{bmatrix}, \tag{2.1b}$$

where $x_{ij}^h \in \Re^{n_1}$ and $x_{ij}^v \in \Re^{n_2}$ are the horizontal and vertical state vectors, $u_{ij} \in \Re^m$ is the input vector and $y_{ij} \in \Re^p$ is the output vector and $A_{11} \in \Re^{n_1 \times n_1}$, $A_{12} \in \Re^{n_1 \times n_2}$, $A_{21} \in \Re^{n_2 \times n_1}$, $A_{22} \in \Re^{n_2 \times n_2}$, $B_1 \in \Re^{n_1 \times m}$, $B_1 \in \Re^{n_2 \times m}$, $C_1 \in \Re^{p \times n_1}$, $C_2 \in \Re^{p \times n_2}$.

Boudary conditions for the model are given by

$$x_{0j}^h \in \Re^{n_1},\ j = 0,1,\ldots,\ x_{i0}^v \in \Re^{n_2},\ i = 0,1,\ldots. \tag{2.1c}$$

Theorem 2.1. The solution of (2.1a) with boundary conditions (2.1c) is given by

$$\begin{bmatrix} x_{ij}^h \\ x_{ij}^v \end{bmatrix} = \sum_{p=0}^{i} T_{i-p,j}\begin{bmatrix} 0 \\ x_{p0}^v \end{bmatrix} + \sum_{q=0}^{j} T_{i,j-q}\begin{bmatrix} x_{0q}^h \\ 0 \end{bmatrix} + \\ + \sum_{p=0}^{i-1}\sum_{q=0}^{j} T_{i-p-1,j-q}\begin{bmatrix} B_1 \\ 0 \end{bmatrix} u_{pq} + \sum_{p=0}^{i}\sum_{q=0}^{j-1} T_{i-p,j-q-1}\begin{bmatrix} 0 \\ B_2 \end{bmatrix} u_{pq} \tag{2.2a}$$

where

$$T_{ij} = \begin{cases} I_n & for \quad i = j = 0 \\ A_{10}T_{i-1,j} + A_{01}T_{i,j-1} \end{cases},\ i,j \ge 0, \tag{2.2b}$$

$$B_{10} = \begin{bmatrix} B_1 \\ 0 \end{bmatrix},\ B_{01} = \begin{bmatrix} 0 \\ B_2 \end{bmatrix} A_{10} = \begin{bmatrix} A_{11} & A_{12} \\ 0 & 0 \end{bmatrix}, A_{01} = \begin{bmatrix} 0 & 0 \\ A_{21} & A_{22} \end{bmatrix}. \tag{2.2c}$$

$$T_{ij} = 0 \text{ for } i < 0 \text{ or/and } j < 0. \tag{2.2d}$$

Definition 2.1. [6-8.9.10,11] The Roesser model (2.1) is called (locally) controllable in the rectangle $[(0,0),(h,k)]$ if for every boundary conditions $x_{0j}^h \in \Re^{n_1}$, $j \in [0,k]$, $x_{i0}^v \in \Re^{n_2}$, $i \in [0,h]$ and every given vector $v_f \in \Re^n$, $(n = n_1 + n_2)$ there exists a sequence of inputs $u_{ij} \in \Re^m$, for $(0,0) \le (i,j) < (h,k)$ such that $x(h,k) = v_f$.

Theorem 2.2. [6-8,9,10,11] The Roesser model is controllable in the rectangle $[(0,0),(h,k)]$ if and only if

$$\mathrm{rank}[M(0,1),\ M(1,0),\ldots,\ M(i,\ j),\ldots,\ M(h,\ k)] = n, \tag{2.3a}$$

where

$$M(i,\ j) = T_{i-1,\ j}B_{10} + T_{i,\ j-1}B_{01}. \tag{2.3b}$$

Definition 2.2. [6] The Roesser model (2.1) is called (locally) observable in the rectangle $[(0,0),(h,k)]$ if there is no local initial state $x(0,0)=0$ such that for zero inputs $u_{ij}=0,$ $(0,\ 0)\le(i,\ j)<(h,\ k)$ and zero boundary conditions $x_{0j}^{h}=0,\ j\in[0,\ k],\ x_{i0}^{v}=0,$ $i\in[0,\ h]$, the output is also zero $y_{ij}=0,$ for $(0,\ 0)\le(i,\ j)<(h,\ k)$.

Theorem 2.3. [6] The Roesser model (2.1) is observable in the rectangle $[(0,0),(h,k)]$ if and only if

$$\mathrm{rank}\begin{bmatrix} C \\ CT_{10} \\ CT_{01} \\ \vdots \\ CT_{hk} \end{bmatrix} = n \tag{2.4}$$

The observability is the dual notion with respect to the local controllability of the Roesser model [10,11].

3. STABILITY OF THE 2D DISCRETE-TIME ROESSER MODEL

Consider the discrete-time autonomous (B=0) model (2.1).

Definition 3.1.[6]. The 2D Roesser model (2.1a) is called asymptotically stable if for zero input $u_{ij}=0,$ and every (bounded) boundary conditions $x_{0j}^{h},\ x_{i0}^{v}$

$$\lim_{i,j\to\infty}\begin{bmatrix} x_{0j}^{h} \\ x_{i0}^{v} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}. \tag{3.1}$$

Consider the autonomous discrete-time Roesser model with

$$\begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} = \begin{bmatrix} a_{11} & \dots & a_{1,n_1} & a_{1,n_{1+1}} & \dots & a_{1,n} \\ \dots & \dots & \dots & \dots & \dots & \dots \\ a_{n_1,1} & \dots & a_{n_1 n_1} & a_{n_1,n_1+1} & \dots & a_{n_1,n} \\ a_{n_1+1,1} & \dots & a_{n_1+1,n_1} & a_{n_1+1,n_1+1} & \dots & a_{n_1+1,n} \\ \dots & \dots & \dots & \dots & \dots & \dots \\ a_{n,1} & \dots & a_{n,n_1} & a_{n,n_1+1} & \dots & a_{n,n} \end{bmatrix} \tag{3.2}$$

For each row of the matrix (3.2) we may define the circle C_{rk} with the center in the point a_{kk}, $k=1,\dots,n$ and the radius

$$R_i = \sum_{\substack{j=1 \\ j\neq i}}^{n} |a_{ij}|, \quad i=1,\dots,n \tag{3.3a}$$

Similarly for each column of the matrix (3.2) we may define the circle C_{ck} with the center in the point a_{kk}, $k=1,\dots,n$ and the radius

$$\overline{R}_i = \sum_{\substack{i=1 \\ j\neq i}}^{n} |a_{ij}|, \quad j=1,\dots,n\,. \tag{3.3b}$$

Theorem 3.1. The Roesser model with (3.2) is asymptotically stable if one of the conditions is satisfies

1) All circles C_{rk} for k=1,...,n are located in the unit circle (with center in the point (0,0) and redius equal 1)
2) All circles C_{ck} for $k=1,\dots,n$ are located in the unit circle.

Proof. Let z_{ki}, $k=1,2;\ i=1,\dots,n_k$ be the eigenvalues of the matrix A_{kk} and v_{ki} the corresponding vector. Then from the equalities

$$A_{kk}v_{ki} = z_{ki}v_{ki} \text{ for } k=1,2;\ \ i=1,\dots,n_k \tag{3.4}$$

we have

$$(z_{ki} - a_{ii})v_{ki} = \sum_{\substack{j=1 \\ j\neq i}}^{n} a_{ij}v_{ki} \text{ for } k=1,2;\ \ i=1,\dots,n_k \tag{3.5}$$

and

$$|z_{ki} - a_{ii}||v_{ki}| \le \sum_{\substack{j=1 \\ j \ne i}}^{n} |a_{ij}||v_{ki}| = R_i |v_{ki}|. \tag{3.6}$$

From (3.6) it follows that the eigenvalues z_{ki} are located inside the circle or on the circle $C_{rk}(C_{ck})$, $k = 1,...,n$. Considerations for columns are similar. This completes the proof. □

Example 3.1. Check the stability of the Roesser model (3.2) with the matrix

$$A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} = \begin{bmatrix} -0.3 & 0.1 & 0.2 & 0.3 \\ 0.2 & -0.1 & 0.2 & 0.2 \\ 0.1 & 0.2 & -0.4 & 0.2 \\ 0.3 & 0.1 & 0.2 & -0.3 \end{bmatrix}. \tag{3.7}$$

Applying the Theorem 3.1 for circles C_{rk} we obtain:

C_{r1} - the center in the point $a_{11} = -0.3$ and radius $R_1 = 0.6$,
C_{r2} - the center in the point $a_{22} = -0.1$ and radius $R_2 = 0.6$,
C_{r3} - the center in the point $a_{33} = -0.4$ and radius $R_3 = 0.5$,
C_{r4} - the center in the point $a_{44} = -0.3$ and radius $R_4 = 0.6$.

Therefore, the first condition of Theorem 3.1 is satisfied.

Applying the Theorem 3.1 for circles C_{ck} we obtain:

C_{c1} - the center in the point $a_{11} = -0.3$ and radius $\overline{R}_1 = 0.6$,
C_{c2} - the center in the point $a_{22} = -0.1$ and radius $\overline{R}_2 = 0.4$,
C_{c3} - the center in the point $a_{33} = -0.4$ and radius $\overline{R}_3 = 0.6$,
C_{c4} - the center in the point $a_{44} = -0.3$ and radius $\overline{R}_4 = 0.7$.

Therefore, the second condition of Theorem 3.1 is satisfied and the Roesser model is asymptotically stable.

Remark 3.1. It is well-known that the similarity transformation

$$\overline{A} = PAP^{-1} \tag{3.8}$$

does not change the eigenvalues of the matrix *A* since

$$\det[I_n z - \overline{A}] = \det[I_n z - A]. \tag{3.9}$$

By suitable choice of the diagonal entries $p_i = 0, \quad i = 1,\dots,n$ of the matrix

$$P = \mathrm{diag}[p_1,\dots, p_n] \tag{3.10}$$

we may obtain the matrix

$$\overline{A} = PAP^{-1} = \begin{bmatrix} a_{11} & \frac{p_1}{p_2}a_{12} & \dots & \frac{p_1}{p_n}a_{1n} \\ \frac{p_2}{p_1}a_{21} & a_{22} & \dots & \frac{p_2}{p_n}a_{2n} \\ \dots & \dots & \dots & \dots \\ \frac{p_n}{p_1}a_{n1} & \frac{p_n}{p_2}a_{n2} & \dots & a_{nn} \end{bmatrix} \tag{3.11}$$

which satisfies the conditions of Theorem 3.1.

Example 3.2. Consider the Roesser model (3.2a) with the matrix

$$A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} = \begin{bmatrix} -0.3 & 0.3 & 0.2 & 0.3 \\ 0.2 & -0.1 & 0.1 & 0.1 \\ 0.1 & 0.2 & -0.3 & 0.2 \\ 0.1 & 0.1 & 0.1 & -0.2 \end{bmatrix} \tag{3.12}$$

In this case the circles for (3.12) have the parameters :

C_{r1} - the center in the point $a_{11} = -0.3$ and radius $R_1 = 0.8$,

C_{r2} - the center in the point $a_{22} = -0.1$ and radius $R_2 = 0.4$,

C_{r3} - the center in the point $a_{33} = -0.3$ and radius $R_3 = 0.5$,

C_{r4} - the center in the point $a_{44} = -0.2$ and radius $R_4 = 0.3$.

In this case the row conditions of Theorem 3.1 are not satisfied since the sum of moduli of the first row of the matrix *A* is greater 1and the condition 1 of Theorem 1 is not satisfied.

For columns the circles for (3.12) have the parameters:

C_{c1} - the center in the point $a_{11} = -0.3$ and radius $\overline{R}_1 = 0.4$,

C_{c2} - the center in the point $a_{22} = -0.1$ and radius $\overline{R}_2 = 0.6$,

C_{c3} - the center in the point $a_{33} = -0.3$ and radius $\overline{R}_3 = 0.4$,

C_{c4} - the center in the point $a_{44} = -0.2$ and radius $\overline{R}_4 = 0.6$.

Note that the sum of modules of the entries of the entries of all columns are less than 1 and the condition 2 of Theorem 3.1 is satisfied. Therefore, the Roesser model with the matrix (3.7) is asymptotically stable.

Following Remark 3.1 we choose the matrix *P* in the form

$$P = \mathrm{diag}[1,1,1,2] \tag{3.13}$$

Using (3.11), (3.12) and (3.13) we obtain

$$\overline{A} = PAP^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} -0.3 & 0.3 & 0.2 & 0.3 \\ 0.2 & -0.1 & 0.1 & 0.1 \\ 0.1 & 0.2 & -0.3 & 0.2 \\ 0.1 & 0.1 & 0.1 & -0.2 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0.5 \end{bmatrix}$$
$$= \begin{bmatrix} -0.3 & 0.3 & 0.2 & 0.15 \\ 0.2 & -0.1 & 0.1 & 0.05 \\ 0.1 & 0.2 & -0.3 & 0.1 \\ 0.2 & 0.2 & 0.2 & -0.2 \end{bmatrix}. \tag{3.14}$$

Note that the matrix (3.14) satisfies both conditions of Theorem 3.1 and the matrix is asymptotically stable.

4. STABILITY OF THE 2D CONTINUOUS-TIME ROESSER MODEL

Consider the continuous-time 2D Roesser model [15,7]

$$\begin{bmatrix} \dfrac{\delta x^h(t_1,t_2)}{\delta t_1} \\ \dfrac{\delta x^h(t_1,t_2)}{\delta t_2} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} x^h(t_1,t_2) \\ x^h(t_1,t_2) \end{bmatrix} + \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} u(t_1,t_2), \tag{4.1a}$$

$$y(t_1,t_2) = [C_1 \quad C_2] \begin{bmatrix} x^h(t_1,t_2) \\ x^h(t_1,t_2) \end{bmatrix}, \tag{4.1b}$$

where $x^h(t_1,t_2) \in \Re^{n_1}$ is the horizontal state vector, $x^v(t_1,t_2) \in \Re^{n_2}$ is the vertical state vector, $u(t_1,t_2) \in \Re^m$ is the input vector, $y(t_1,t_2) \in \Re^p$ is the output vector, $A_{11} \in \Re^{n_1 \times n_1}$, $A_{12} \in \Re^{n_1 \times n_2}$, $A_{21} \in \Re^{n_2 \times n_1}$, $A_{22} \in \Re^{n_2 \times n_2}$, $B_1 \in \Re^{n_1 \times m}$, $B_1 \in \Re^{n_2 \times m}$, $C_1 \in \Re^{p \times n_1}$,

$C_2 \in \Re^{p \times n_2}$ are real matrices.

Boundary conditions for the model are given by

$$x^h(0,t_2) \in \Re^{n_1},\ x^v(t_1,0) \in \Re^{n_2} \text{ for } t_1, t_2 \geq 0. \tag{4.1c}$$

The solution of (4.1a) with boundary conditions (4.1c) is given in15].

Definition 4.1. The continuous-time Roesser model (4.1) is called asymptotically stable if for zero input $u(t_1,t_2) = 0$ and every bounded boundary conditions $x^h(0,t_2) \in \Re^{n_1}$, $x^v(t_1,0) \in \Re^{n_2}$

$$\lim_{t_1,t_2 \to \infty} \begin{bmatrix} x^h(t_1,t_2) \\ x^h(t_1,t_2) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \tag{4.2}$$

Consider the autonomous Roesser model

$$\begin{bmatrix} \dfrac{\delta x^h(t_1,t_2)}{\delta t_1} \\ \dfrac{\delta x^h(t_1,t_2)}{\delta t_2} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} x^h(t_1,t_2) \\ x^h(t_1,t_2) \end{bmatrix}, \tag{4.3a}$$

where

$$\begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} = \begin{bmatrix} a_{11} & \cdots & a_{1,n_1} & a_{1,n_{1+1}} & \cdots & a_{1,n} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ a_{n_1,1} & \cdots & a_{n_1 n_1} & a_{n_1,n_1+1} & \cdots & a_{n_1,n} \\ a_{n_1+1,1} & \cdots & a_{n_1+1,n_1} & a_{n_1+1,n_1+1} & \cdots & a_{n_1+1,n} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ a_{n,1} & \cdots & a_{n,n_1} & a_{n,n_1+1} & \cdots & a_{n,n} \end{bmatrix}. \tag{4.3b}$$

Theorem 4.1. The 2-D continuous-time Roesser model (4.3) is asymptotically stable if the diagonal entries of the matrix (4.4b) are negative and one of the following conditions

$$|a_{kk}| > \sum_{\substack{i=1 \\ i \neq k}}^{n} |a_{ki}|, \quad k = 1,\dots,n \tag{4.4a}$$

or

$$\left|a_{kk}\right| > \sum_{\substack{j=1 \\ j\neq k}}^{n} \left|a_{jk}\right|, \quad k = 1,\dots,n \tag{4.4b}$$

is satisfied.

Proof. Let s_{ki}, $k=1,2;\ \ i=1,\dots,n_k$ be the eigenvalues of the matrix A_{kk} and w_{ki} the corresponding vectors. Then from the equalities

$$A_{kk} w_{ki} = s_{ki} w_{ki} \text{ for } k=1,2; \ \ i=1,\dots,n_k \tag{4.5}$$

we have

$$(s_{ki} - a_{ii}) w_{ki} = \sum_{\substack{j=1 \\ j\neq i}}^{n} a_{ij} w_{ki} \text{ for } k=1,2; \ \ i=1,\dots,n_k \tag{4.6}$$

and

$$\left|s_{ki} - a_{ii}\right| \left|w_{ki}\right| \le \sum_{\substack{j=1 \\ j\neq i}}^{n} \left|a_{ij}\right| \left|w_{ki}\right| = \left|R_i\right| \left|w_{ki}\right|. \tag{4.7}$$

From (4.8) it follows that the eigenvalues s_{ki} are located inside the circle or on the circle $C_{rk}(C_{ck})$, $k=1,\dots,n$. The considerations for rows are similar. This completes the proof.

Example 4.1.Check the stability of the Roesser model (4.3) with the matrix

$$\begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} = \begin{bmatrix} -2 & 0.8 & 0.3 & 0.4 \\ 1 & -3 & 0.8 & 0.6 \\ 0.4 & 0.6 & -2 & 0.8 \\ 0.3 & 0.2 & 0.4 & -2.4 \end{bmatrix} \tag{4.8}$$

Using Theorem 4.1 we obtain

C_{r1} - the center in the point $a_{11} = -2$ and radius $R_1 = 1.5$,

C_{r2} - the center in the point $a_{22} = -3$ and radius $R_2 = 2.4$,

C_{r3} - the center in the point $a_{33} = -2$ and radius $R_3 = 1.8$,

C_{r4} - the center in the point $a_{44} = -2.4$ and radius $R_4 = 0.9$.

Note that the row conditions of Theorem 4.1 are satisfied.

For columns the circles for (4.9) have the parameters:

C_{c1} - the center in the point $a_{11} = -2$ and radius $\overline{R}_1 = 1.7$,

C_{c2} - the center in the point $a_{22} = -3$ and radius $\overline{R}_2 = 1.6$,

C_{c3} - the center in the point $a_{33} = -2$ and radius $\overline{R}_3 = 1.5$,

C_{c4} - the center in the point $a_{44} = -2.4$ and radius $\overline{R}_4 = 1.8$.

The column conditions of Theorem 4.1 are also satisfied. Therefore, the Roesser model with (4.8) is asymptotically stable.

5. CONCLUDING REMARKS

New simple sufficient conditions for the asymptotic stability of the discrete-time and continuous-time 2D Roesser models have been proposed. The effectiveness of the conditions has been demonstrated by simple numerical examples. The presented new stability conditions can be extended to the Fornasini-Marchesini models and the general model of 2D linear systems. An open problem is an extension of these considerations to the fractional orders 2D linear systems.

BIBLIOGRAPHY

[1] Gantmacher F.R., The Theory of Matrices. Chelsea Pub. Comp., London, 1959

[2] Fornasini E.and Marchesini G.,Doubly indexed dynamical systems,state space models and structural properties,Mathematical Systems Theory,vol.12.no.1,1978.

[3] Kaczorek T. Linear Control Systems, vol. 1 and 2,Reaserch Studies Press LTD, J.Wiley, New York 1992.

[4] Kaczorek T., Positive switched 2D linear systems described by the Roesser models, European Journal of Control vol.3.2012,1-8.

[5] Kaczorek T.Positive switched 2D linear systems described by the Roesser models,Proc.19th Symp.Math.Theory of Networks and Systems. 2010,5-9 July,Budapest.

[6] Kaczorek T.,Two-Dimensional Linear Systems, Springer-Verlag, Berlin 1985,

[7] Kaczorek T and Rogowski K., Fractional Linear Systems and Electrical Circuits, Springer, Switzerland 2015.

[8] Kalman R.E., On the general theory of control systems, Proceedings of the IFAC Congress Automatic Control, zp. 481-492, 1960.

[9] Kalman R.E., Mathematical description of linear dynamical systems, SIAM Journal of Control, Series A, 152-192, 1963.

[10] Klamka J., Controllability of Dynamical Systems, Kluwer Academic Publishers, Dordrecht 1991.

[11] Klamka J., Controllability and Minimum Energy Control, Studies in Systems, Decision and Control. vol.162. Springer Verlag 2018.

[12] Kurek J.,The general state space model for a two-dimensional linear digital systems,IEEE Trans.Autom.Contr.,June 1985,AC-30,no.6.600-601.

[13] Mitkowski W., Outline of Control Theory, Publishing House A.H, Krakow, 2019.

[14] Roesser R.P.,A discrete state-space model for linear image processing.IEEE Trans.Autom.Contr. Ac-20,1-10,1975.

[15] Rogowski K.,General response formula for fractional 2D continuous-time linear systems described by the Roesser model, Acta Mechanica et Automatica vol5,no2,2011,112-116.

[16] Sajewski L., Stabilization of positive descriptor fractional discrete-time linear system with two different fractional orders by decentralized controller, Bull. Pol. Ac.: 65(5), 709-714, 2017.

[17] Zak, S., Systems and Control, Oxford University Press, New York 2003.