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MULTI-CRITERIA MULTI-STAGE GAME OPTIMIZATION

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Abstract – The article presents a mathematical model of a multi-stage game of the process of safe control of a transport object in possible collision situations with other encountered objects, containing a description of state variables, state and control constraints, and sets of permissible object strategies. Multi-criteria optimization tasks were formulated in the form of positional and matrix games under the conditions of playing non-cooperative and cooperative control as well as non-game optimal control. The multi-criteria control algorithms corresponding to these tasks were computer simulated in Matlab / Simulink on the example of a real situation.

Key words – optimization, control engineering, game theory, computer simulation

INTRODUCTION

The issues of optimal control of transport and logistics processes can be divided into those for which the cost of the process: is an unambiguous control function, depends on the control method and on some random event with a known statistical description, or is determined by the choice of the control method and some undefined factor. The last group of issues concerns transport and logistic game processes, the synthesis of which is carried out using the methods of game theory [1-4,18]. The basic game control systems are positional control systems of objects as feedback systems representing positional and matrix games, for example, safe steering of a ship in collision situations at sea.[1]

I. MODEL OF THE MULTI-STAGE GAME PROCESS

The essence of the positional game is the dependence of the strategy of one's own object on the position $p(t_k)$ of the encountered objects in the current stage of motion k. In this way, possible changes in the course and speed of the encountered objects during the control are taken into account in the process model [6-10] (Fig. 1).

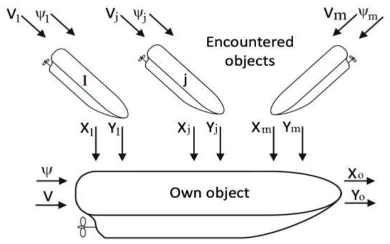

Fig. 1. Schematic diagram of the control process of moving objects

I.1. The state of the control process

The current state of the process is determined by the x_0 position of the own object and the x_j positions of the encountered objects.

$$x_0 = (X_0, Y_0),\ x_j = (X_j, Y_j),\ j = 1, 2, \ldots, m \tag{1}$$

The system generates its control at the moment of t_k based on the data it receives from the radar anti-collision system about the current position of the tracked objects:

$$p(t_k) = \begin{bmatrix} x_0(t_k) \\ x_j(t_k) \end{bmatrix} \ j = 1,2,\ldots,m \ k = 1,2,\ldots,K \tag{2}$$

It is assumed, in accordance with the general concept of multi-stage games, that the position of the objects encountered is known at every discrete time moment t_k on one's own object [12,17,20,27-31].

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I.2. The state and control variables constraints

The state variable constraints are the navigational limitations:

$$\left\{ x_0(t),\, x_j(t) \right\} \in P \tag{3}$$

The control constraints take into account the kinematics of the movement of objects, the recommendations of the right of way and the condition of maintaining a safe passing distance d_s (Figure 2) [15]:

$$u_0 \in U_0, u_j \in U_j, j = 1, 2, ..., m \tag{4}$$

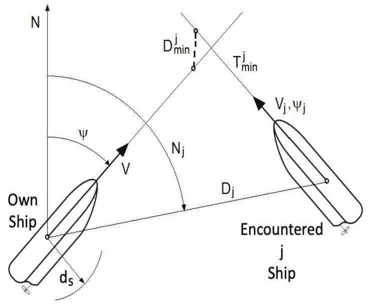

Fig. 2. State variables describing the process of passing ships

I.3. The sets of allowed objects strategies

The sets of permissible strategies of the game participants in relation to each other are dependent, which means that the selection of the control of the j by the j-th encountered object changes the sets of permissible strategies of other objects described by the relationship:

$$\begin{aligned} U_{0,j}[p(t)] &= S_{01,j} \cup S_{02,j} \\ U_{j,0}[p(t)] &= S_{j1,0} \cup S_{j2,0} \end{aligned} \tag{5}$$

Figure 3 shows the method of determining the sets of acceptable object strategies - own 0 and met j.

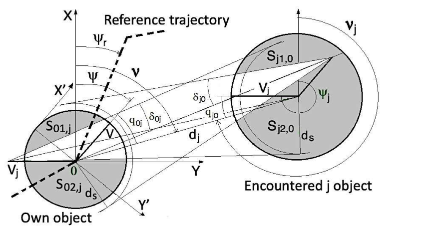

Fig. 3. Determining sets of permissible strategies of the own object 0 and the encountered object j

The set $U_{0,j}$ is defined by the inequalities:

$$\begin{aligned} & f_{0,j}\, u_{0,x'} + g_{0,j}\, u_{0,y'} \le h_{0,j} \\ & u^2_{0,x'} + u^2_{0,y'} \le V^2 \end{aligned} \tag{6}$$

where:

$$\begin{aligned} & \vec{V} = \vec{u}_0 (u_{0,x'}, u_{0,y'}) \\ & f_{0,j} = -\lambda_{0,j} \cos(q_{0,j} + \lambda_{0,j}\, \delta_{0,j}) \\ & g_{0,j} = \lambda_{0,j} \sin(q_{0,j} + \lambda_{0,j}\, \delta_{0,j}) \\ & h_{0,j} = -\lambda_{0,j} \begin{bmatrix} V_j \sin(q_{j,0} + \lambda_{0,j}\, \delta_{0,j}) + \\ V \cos(q_{0,j} + \lambda_{0,j}\, \delta_{0,j}) \end{bmatrix} \\ & w_{0,j} = \begin{cases} -1 & \text{dla } S_{01,j} \quad (LB) \\ 1 & \text{dla } S_{02,j} \quad (PB) \end{cases} \end{aligned} \tag{7}$$

The value of w_{0j} is determined by the logical function W_j characterizing the requirements of the rules of the right of way. The form of the W_j function depends on the interpretation of the right of way rules for the synthesis of the safe object control algorithm:

$$W_j = \begin{cases} 1 \; to \; w_{0,j} = 1 \\ 0 \; to \; w_{0,j} = -1 \end{cases} \tag{8}$$

The set of acceptable strategies of the j-th object for the own object is determined analogously by determining the following inequalities:

$$\begin{aligned} & f_{j,0}\, u_{j,x'} + g_{j,0}\, u_{j,y'} \le h_{j,0} \\ & u^2_{j,x'} + u^2_{j,y'} \le V_j^2 \end{aligned} \tag{9}$$

where:

$$\begin{aligned} & \vec{V}_j = \vec{u}_j (u_{j,x'}, u_{j,y'}) \\ & f_{j,0} = -\lambda_{j,0} \cos(q_{j,0} + \lambda_{j,0}\, \delta_{j,0}) \\ & g_{j,0} = \lambda_{j,0} \sin(q_{j,0} + \lambda_{j,0}\, \delta_{j,0}) \\ & h_{j,0} = -\lambda_{j,0} V \sin(q_{0,j} + \lambda_{j,0}\, \delta_{j,0}) \\ & w_{j,0} = \begin{cases} -1 & \text{dla } S_{j1,0} \quad (LB) \\ 1 & \text{dla } S_{j2,0} \quad (PB) \end{cases} \end{aligned} \tag{10}$$

II. MULTI-CRITERIAL OPTIMIZATION OF THE GAME

To determine the optimal maneuver of the own object in the allowable control area, which is connected to all objects encountered:

$$U_o = \bigcap_{j=1}^{m} U_{0,j} \quad j = 1, 2, ..., m \tag{11}$$

Optimal control of own object $u_0^*(t)$, equivalent to the current position p(t) optimal positional control $u_0^*(p)$, is determined as follows:

- sets of acceptable strategies $U_{j,0}[p(t_k)]$ of the encountered objects in relation to the own object and the initial sets $U_{0,j}[p(t_k)]$ of acceptable strategies of the own object in relation to each of the encountered objects are defined,
- with respect to each j-th encountered object, a pair of vectors $u_{0,j}$ and $u_{j,0}$ are determined, and then the optimal positional strategy of the own object from the optimum condition Q* of the control quality index [5,13,14,16,23-26].

II.1. Non-cooperative positional game control algorithm pgnc

The pgnc multi-stage positional game algorithm uses the following optimization criterion:

$$Q^*_{pgnc} = \min_{u_0^* \in U_0 = \bigcap_{j=1}^{m} U_{0,j}} \left\{ \max_{u_{j,0} \in U_{j,0}} \min_{u_{0,j} \in U_{0,j}(u_j)} L\left[x_0(t_k), L_k\right] \right\} = L^*_{0,nc}$$
$$j = 1,2,\ldots,m \tag{12}$$

where: L_0 - means the ship's own continuous steering target function, characterizing the ship's distance at time t_0 to the nearest turning point L_k on a given voyage route.

First, the control of own ship is determined, ensuring the shortest passing trajectory, i.e. the smallest path losses (condition min) for non-cooperative steering of each ship that meets, contributing to the greatest extension of the own ship's trajectory (condition max).

Finally, from the own ship's controls set to the individual j ships met, the own ship controls are selected for all n encountered ships, ensuring the lowest path losses (condition min).

According to the three optimization conditions (min max min), a triple linear programming method is used to solve the game, obtaining the optimal values for the course and speed of your own ship.

The smallest loss of the road is achieved for the maximum projection of the own ship's speed vector on the direction of the set course. The optimal control is calculated many times at each discrete stage of motion using the Simplex method to solve the linear programming problem for variables in the form of the ship's own velocity vector components.

II.2. Cooperative positional game control algorithm pgc

The pgc multi-stage positional cooperative game algorithm uses the following optimization criterion:

$$Q^*_{pgc} = \min_{u_0^* \in U_0 = \bigcap_{j=1}^{m} U_{0,j}} \left\{ \min_{u_{j,0} \in U_{j,0}} \min_{u_{0,j} \in U_{0,j}(u_j)} L\left[x_0(t_k), L_k\right] \right\} = L^*_{0,c}$$
$$j = 1,2,\ldots,m \tag{13}$$

The difference from the pgnc algorithm results from the use of cooperative action between ships in order to avoid collisions by all j objects and replacing the second condition max to min.

II.3. Non-game positional control algorithm ngpc

The ngpc multi-stage non-game control algorithm uses the following optimization criterion:

$$Q^*_{ngpc} = \min_{u_0^* \in U_0 = \bigcap_{j=1}^{m} U_{0,j}} \left\{ L\left[x_0(t_k), L_k\right] \right\} = L^*_{0,oc}$$
$$j = 1,2,\ldots,m \tag{14}$$

The choice of the optimal trajectory of the ship according to criteria (12), (13) and (14) comes down to determining its course and speed ensuring the least loss of the way to safely pass the encountered objects, at a distance not shorter than the assumed value of d_s, taking into account the dynamics of the ship in the form of advance maneuver time. The smallest loss of the road is achieved for the maximum projection of the own ship's speed vector on the direction of the set course. The lead time consists of the lead time and the lead time of the own vessel's speed change [19].

II.4. Non-cooperative risk game control algorithm rgnc

The risk value (15) is possible to define by referring the current situation of approach, described by parameters D^j_{min} and T^j_{min}, to the assumed evaluation of the situation as safe, determined by a safe distance of approach d_s and a safe time T_s – which are necessary to execute a collision avoiding manoeuvre with consideration of distance D_j to j-th met ship:

$$r_j = \left[k_1 \left(\frac{D^j_{\min}}{D_s} \right)^2 + k_2 \left(\frac{T^j_{\min}}{T_s} \right)^2 + \left(\frac{D_j}{D_s} \right)^2 \right]^{-\frac{1}{2}} \tag{15}$$

The weight coefficients k_1 and k_2 depended on the state visibility at sea, dynamic length L_d and dynamic beam B_d of the ship, kind of water region and in practice are equal:

$$0 \le [k_1(L_d, B_d), k_2(L_d, B_d)] \le 1 \tag{16}$$

$$L_d = 1.1\ (1 + 0.345\ V^{1.6}) \tag{17}$$

$$B_d = 1.1\ (B + 0.767\ LV^{0.4}) \tag{18}$$

As a result of using the following form for the control goal:

$$Q^*_{rgnc} = \min_{u_o} \max_{u_j} r_j \tag{19}$$

the probability matrix $\mathbf{P}=[p_j\ (u_0,u_j)]$ of using particular pure strategies may be obtained.

The solution for the control problem is the strategy representing the highest probability:

$$u_o^* = u_o \left\{ [p_j(u_o, u_j)]_{max} \right\} \tag{20}$$

II.5. Cooperative risk game control algorithm rgc

The quality index of control for a cooperative game has the form:

$$Q^*_{rgc} = \min_{u_o} \min_{u_j} r_j \tag{21}$$

II.6. Non-game risk control algorithm ngrc

In the usual case of non-game control, the quality index is reduced to the following form:

$$Q^*_{ngrc} = \min_{u_o} r_j \tag{22}$$

The pgnc, pgc, ngpc, rgnc, rgc and ngrc algorithms for determining the safe trajectory of the ship in a collision situation were developed using the lp - linear programming function from the Optimization Toolbox of the Matlab/Simulink software [11,21,22].

III. COMPUTER SIMULATION OF CONTROL ALGORITHMS

Safe trajectories of own ship in the situation of 19 encountered ships in the Kattegat Strait, in conditions of limited visibility at sea at d_s=1.5 nm (Table 1), determined according to multi-criteria optimization algorithms, are shown in Figs. 4 to trough 11.

Table 1. Movement parameters of the own ship and encountered 19 ships

j	D_j nm	N_j deg	V_j kn	ψ_j deg
0	-	-	20	0
1	9	320	14	90
2	2	10	16	180
3	8	10	15	200
4	12	35	17	275
5	7	270	14	50
6	8	100	8	6
7	11	315	10	90
8	13	325	7	45
9	7	45	19	10
10	15	23	6	275
11	15	23	7	270
12	4	175	4	130
13	13	40	0	0
14	7	60	16	20
15	8	120	12	30
16	9	150	10	25
17	8	310	12	135
18	10	330	10	140
19	9	340	8	150

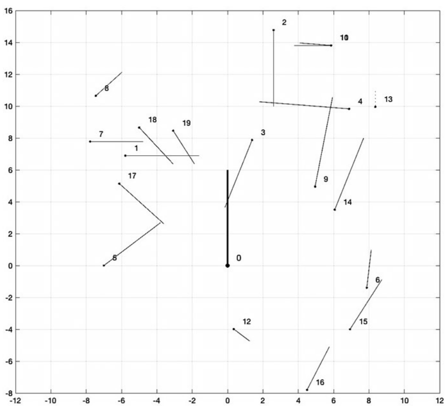

Fig. 4. The eighteen minutes speed vectors of own ship 0 and j=19 encountered ships in navigational situation in Kattegat Strait

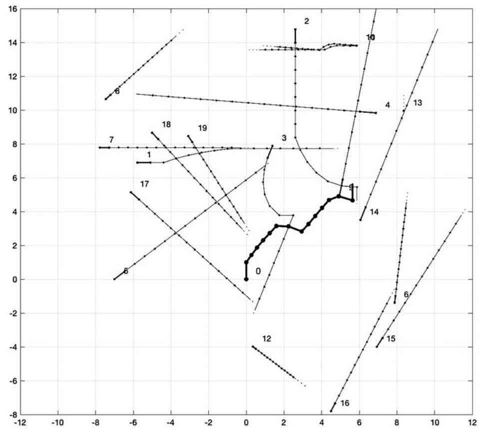

Fig. 5. Ship trajectories in non-cooperative positional game pgnc

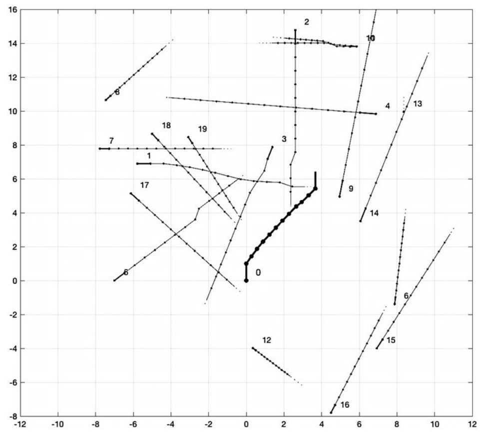

Fig. 6. Ship trajectories in cooperative positional game pgc

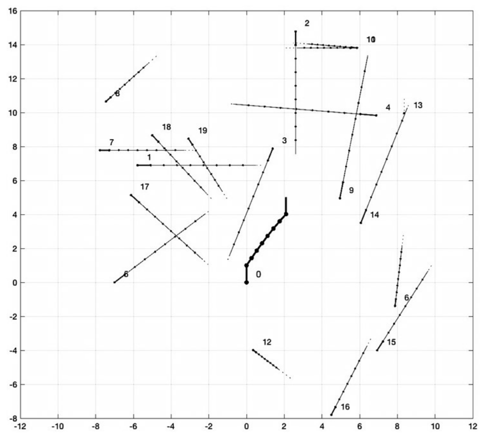

Fig. 7. Ship trajectories in non-game positional control ngpc

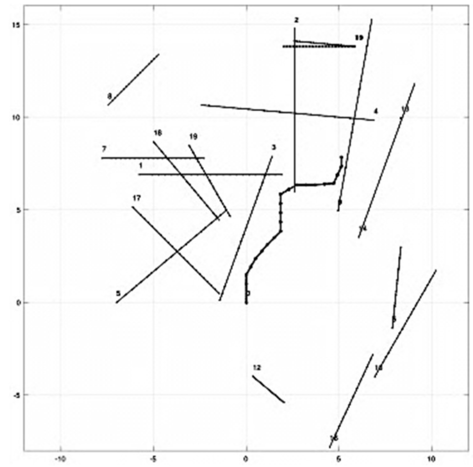

Fig. 8. Ship trajectories in non—cooperative matrix game rgnc

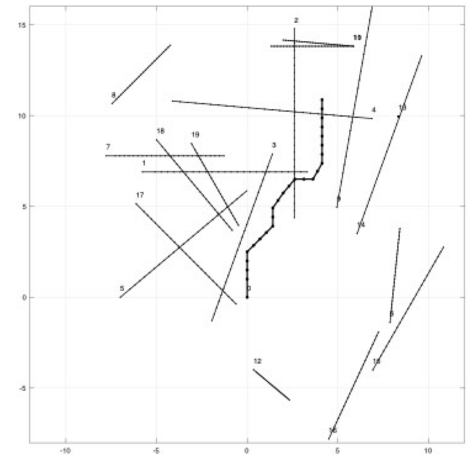

Fig. 9. Ship trajectories in cooperative matrix game rgc

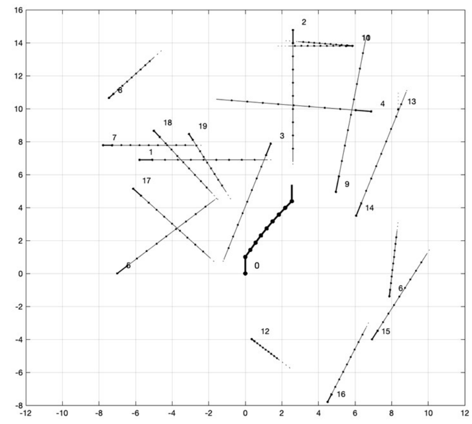

Fig. 10. Ship trajectories in non-game matrix control ngrc

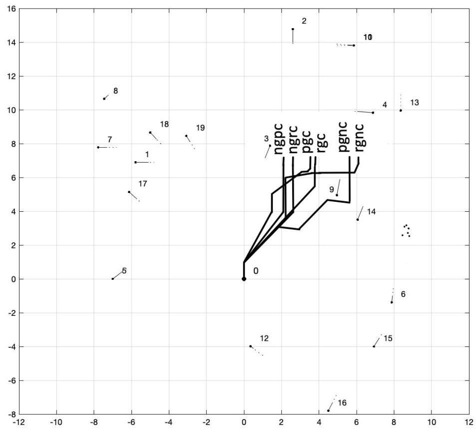

Fig. 11. Comparison of optimal ships trajectories

IV. Conclusions

The purpose of multi-criteria optimization is to determine a set of the best solutions that simultaneously meet a set of many, often contradictory, optimization criteria, for example risk minimization and profit maximization.

Dynamic optimization concerns problems that change over time, i.e. such practical optimization tasks in which the control objective function changes during the transport or production control process.

Taking into account the high complexity of the dynamic game model, simplified models are formulated for the practical synthesis of control algorithms. Particular simplified models of the process are assigned appropriate algorithms for determining the safe trajectory of the ship in situations of excessive proximity with other encountered fixed and movable objects, which can be used in practice for computer-aided maneuvering decisions of the navigator.

Formulating a mathematical model of the process of safe control a moving object while passing a larger number of other moving objects encountered as game models, it is possible to take into account the degree of ambiguity of the situation caused by the imperfection of the right of way rules and the subjectivity of the operator making a maneuvering decision in order to avoid a collision.

The multi-criteria approach to the task of optimizing the control of safe movement of objects allows the synthesis of appropriate algorithms for controlling non-cooperative, cooperative and non-game control.

The obtained safe trajectories differ primarily in the value of the final deviation from the set trajectory of movement, which is a measure of the extension of the object's trajectory and the increase in the cost of its implementation, for example in the form of additional energy consumption of the drive system.

WIELOKRYTERIALNA OPTYMALIZACJA GRY WIELOETAPOWEJ

W artykule przedstawiono model matematyczny wieloetapowej gry procesu bezpiecznego sterowania obiektem transportowym w możliwych sytuacjach kolizyjnych z innymi spotkanymi obiektami, zawierający opis zmiennych stanu, ograniczeń stanu i sterowania oraz zbiory dopuszczalnych strategii obiektów. Sformułowano wielokryterialne zadania optymalizacyjne w postaci gry pozycyjnej i macierzowej, w warunkach rozgrywającego sterowania niekooperacyjnego i kooperacyjnego oraz nierozgrywającego sterowania optymalnego. Algorytmy sterowania wielokryterialnego odpowiadające tym zadaniom poddano symulacji komputerowej w programie Matlab/Simulink na przykładzie rzeczywistej sytuacji.

Słowa kluczowe: optymalizacja, automatyka, teoria gier, symulacja komputerowa

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