



## ANALYSIS OF OPTIMIZATION METHODS IN MARINE OBJECTS CONTROL SYNTHESIS

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**Abstract**—The article presents four main chapters that allow you to formulate an optimization task and choose a method for solving it from static and dynamic optimization methods to single-criterion and multi-criteria optimization. In the group of static optimization methods, the methods are without constraints and with constraints, gradient and non-gradient and heuristic. Dynamic optimization methods are divided into basic - direct and indirect and special. Particular attention has been paid to multi-criteria optimization in single-object approach as static and dynamic optimization, and multi-object optimization in game control scenarios. The article shows not only the classic optimization methods that were developed many years ago, but also the latest in the field, including, but not limited to, particle swarms.

**Key words**—optimization methods, optimal control, safe ship control, computer simulation

### INTRODUCTION

The main goal of optimization is to implement the object control process in the best way. The process can be physical phenomenon, technological process, technical object, economic system, production and transport planning, etc. The mathematical description of the process formulated for the purposes of its optimization is its model. Optimization is as good as the mathematical model is adequate (Fig. 1).

The function  $F(x)$  means the assessment of the quality of the object's operation or the course of the control process and takes the name of the function of the control purpose or control quality index, and  $x$  are the decision variables or variables of the control process state [1-3].

The optimization task consists in determining the values of state variables  $x^*$  at which the control goal function  $F^*(x^*)$  takes the minimum or maximum value [4-6].

The values of the components of the state vector  $x$  cannot be arbitrary and are subject to various restrictions.<sup>1</sup>

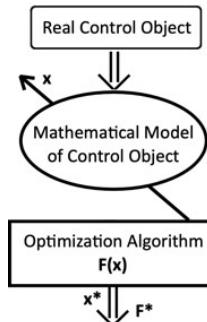


Fig. 1. Stages of formulating and solving the optimization task

### I. FORMULATION OF THE TASK OF OPTIMIZATION

A distinction is made between inequality and equality:

$$g_j(x) \leq 0, j = 1, 2, \dots, m \quad (1)$$

$$h_l(x) = 0, l = 1, 2, \dots, r \quad (2)$$

The introduction of any equality constraint reduces the size of the optimization space by one and may result in the lack of an optimal solution.

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The set of elements of the optimization space satisfying the equation of constraints is called the set of acceptable solutions D:

$$D = \{x : g_j(x) \leq 0, h_i(x) = 0\} \quad (3)$$

The presence of inequality constraints may be the reason why the optimum task does not belong to the set of acceptable solutions.

A general breakdown of optimization methods considered the most representative is shown in Figure 2

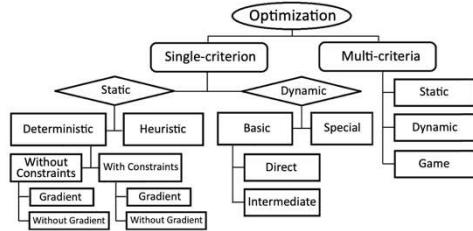


Fig. 2. Categorization of optimization methods

There is a general distinction between static and dynamic optimization tasks. For the synthesis of the optimal controller or the optimal control algorithm for a given object, both static and dynamic optimization methods are used.

In general, each maximization task can be formally reduced to a minimization task, and vice versa, by the relationship:

$$\max F(x) = -\min[-F(x)] \quad (4)$$

The task of **static optimization** is to look for the minimum or maximum output size of the object or its function:

$$F = f(x) \quad (5)$$

while meeting the constraints on the variables x.

The task of **dynamic optimization** is to look for the minimum or maximum functional as integral of the function:

$$F = \int f_o(x, u, t) dt \quad (6)$$

where the dynamic properties of the control object are described by the state equations:

$$\dot{x} = f(x, u, t) \quad (7)$$

$$y = w(x, u, t) \quad (8)$$

and meeting the constraints on state variables x and control quantities u. The object or control process described by the state equations and output can be presented as in Figure 3.

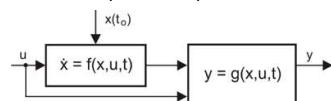


Fig. 3. Optimal control object

At the same time, state variables and their subsequent derivatives are usually taken as the output quantities of the object.

In practice, optimal control of an object or a technological process is implemented in a closed system by an optimal controller or an optimal control algorithm stored in the memory of a microcontroller or a microprocessor-

based PLC programmable controller (Programmable Logic Controller), shown in Figure 4.

Fig. 4. Optimal control system

## II. STATIC OPTIMIZATION

The static optimization task can be formulated as finding the optimal value of the  $x^*$  variable that minimizes or maximizes the goal control function as an optimal control quality index  $F(x)$  in the form of a relationship:

$$F(x) = f(x) \text{ for } x = x_1, x_2, \dots, x_n \quad (9)$$

while meeting the system of equality and inequality constraints:

$$g_j(x) \stackrel{\leq}{\geq} b_j, j = 1, 2, \dots, m \quad (10)$$

Static optimization tasks can be divided into linear and nonlinear programming tasks.

### Linear programming task

$$F = f(x_1, x_2, \dots, x_n) = \sum_{i=1}^n c_i x_i, i = 1, 2, \dots, n \quad (11)$$

$$g(x_1, x_2, \dots, x_n) = \sum_{j=1}^m a_{ij} x_i = b_j, j = 1, 2, \dots, m \quad (12)$$

### Nonlinear programming tasks

#### Case 1

F - nonlinear control purpose function,  
g - nonlinear equality constraints:

$$g_j(x) = b_j \quad (13)$$

as classic optimization problem solved by Lagrange method.

#### Case 2

F - nonlinear control purpose function,

g - linear restrictions:

F as an additive function:

$$F = f(x_1, x_2, \dots, x_n) = f_1(x_1) + f_2(x_2) + \dots + f_n(x_n) \quad (14)$$

F as the sum of the linear and quadratic forms constituting the task of quadratic programming:

$$F = f(x_1, x_2, \dots, x_n) = \sum_{i=1}^n c_i x_i + \sum_{k=1}^s \sum_{l=1}^n d_{ik} x_k x_l \quad (15)$$

F - nonlinear control purpose function,

g - nonlinear restrictions:

$$g_j(x_1, x_2, \dots, x_n) = g_{j1}(x_1) + g_{j2}(x_2) + \dots + g_{jn}(x_n) \quad (16)$$

### II.1. Deterministic methods without constraints without gradient:

1. The golden ratio method,
2. Bisection method,
3. Gauss-Seidel method,
4. Branch and bound method,
5. Branch and cut method,
6. Hooke-Jeeves method,
7. Square interpolation method,
8. Nelder-Mead simplex method,

9. Rosenbrock method
10. Davies-Swann-Campey method.

#### **II.2. Deterministic methods without constraints with gradient:**

1. Simple gradient method,
2. The fastest fall method,
3. Newton-Raphson method,
4. Hestenes-Stiefel conjugate gradient method,
5. Levenberg-Marquardt method,
6. Powell method,
7. Zangwill method.

#### **II.3. Deterministic methods with constraints without gradient:**

1. Lagrange'a method,
2. Linear programming method (Tab. 1),

**Table 1. Example - Optimization of sea container transport**

Ship Port	S1	S2	S3	S4	Containers in Port
P1 Lizbona			1800 TEU		1800 TEU
P2 Le Havre		1000 TEU		1100 TEU	2100 TEU
P3 Bremerhaven	2600 TEU	500 TEU			3100 TEU
P4 Gdańsk		1500 TEU	300 TEU		1800 TEU
P5 Sankt Petersburg		1200 TEU			1200 TEU
<b>Loading ship</b>	2600 TEU	4200 TEU	2100 TEU	1100 TEU	

3. Quadratic programming method,
4. Kuhn-Tucker method,
5. Schmidt-Fox method.

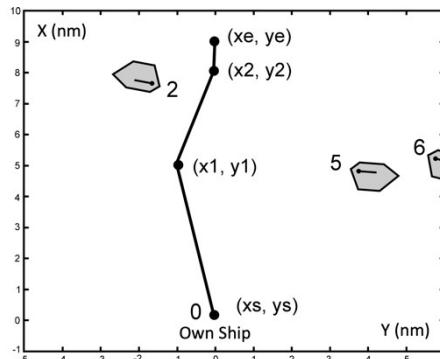
#### **II.4. Deterministic methods with constraints with gradient:**

1. Zoutendijk method,
2. Projected Rosen gradient method.

#### **II.5. Heuristics methods:**

1. Grouping method,
2. Monte Carlo method,
3. Simulated annealing method,
4. Genetic algorithm,
5. Search algorithm,
6. The harmony algorithm,
7. Particle swarm methods:
  - a. Bird's algorithm - Particle Swarm Optimization PSO,
  - b. Base Bees algorithm – BBA,
  - c. Firefly algorithm – FA,
  - d. Cuckoo Search - CS algorithm,
  - e. Cockroach Swarm Optimization - CSO algorithm,
  - f. Flower Pollination - FPA algorithm,
  - g. Cuttlefish algorithm - CFA algorithm,
  - h. Krill Herd - KH algorithm,
  - i. Bat algorithm,

j. Ant Colony Optimization – ACO algorithm (Fig. 5).



**Fig. 5. Determining ship's safe trajectory in collision situation by ACO algorithm [7]**

### **III. DYNAMIC OPTIMIZATION**

The task of dynamic optimization is to look for the minimum or maximum functional as an integral of the function:

$$F = \int_{t_0}^{t_k} f_o(x, u, t) dt \quad (17)$$

The dynamic properties of the control object are described by the state and output equations:

$$\dot{x} = f(x, u, t) \quad (18)$$

$$y = w(x, u, t)$$

with state and control restrictions:

$$g(x) \leq 0 \quad (19)$$

$$h(u) \leq 0$$

The tasks of dynamic optimization can be solved analytically as tasks of time-optimal control and minimizing the objective function in quadratic form, with linear state equations [8].

In contrast, various dynamic optimization tasks in practical applications are most often solved using appropriate numerical methods.

Examples of dynamic optimization tasks in maritime transport:

- determining the optimal route of the ship from the initial port to the destination port ensuring minimum fuel consumption taking into account navigation restrictions and hydrometeorological forecasts,
- determining the optimal anti-collision maneuver of your own ship, ensuring the minimum risk of collision when passing encountered ships,
- optimal ship control at a given course ensuring maximum accuracy and minimum control costs,
- optimization of the ship's main engine control ensuring minimal fuel consumption,
- optimization of the ship loading ensuring maximum ship stability,
- optimization of power distribution to ship propellers

- ensuring maximum ship control,
- optimization of the ship's power system ensuring maximum reliability of supplying the ship's equipment.

### III.1. Basic direct methods:

- Euler variation calculus method,
- Bellman's principle of optimality (Fig. 6),

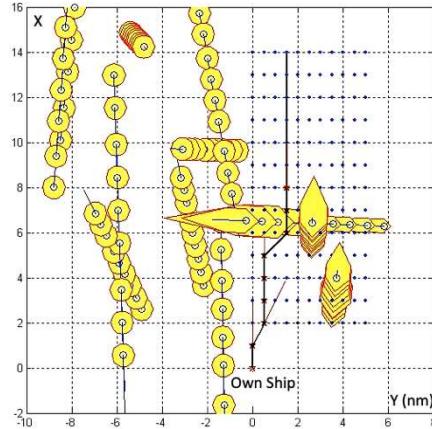


Fig. 6 Determining ship's safe trajectory in collision situation by DYNAMIC PROGRAMMING algorithm

- Simple gradient method in the control space,
- Coupled gradient method in control space,
- Variable metric method,
- The second variation method.

### III.2. Basic indirect methods:

- The principle of maximum Pontriagin,
- Newton's method in the state space,
- Newton-Raphson method in conjugate space.

### III.3. Special methods:

- Neustadt time-optimal control method,
- Gilbert's method,
- Barr's method,
- Balakrishnan penalty functional method,
- Findeisen two-level optimization method.

## IV. MULTI-CRITERIA OPTIMIZATION

### IV.1. Static multi-criteria Pareto optimization

The 80/20 rule, which has been known for a long time, says that 80% of the results are due to only 20% of reasons, in other words, more modest means and less effort can be achieved. The pattern underlying this principle was discovered in 1897 by the Italian economist Vilfredo Pareto [9-12].

The definition of the set of Pareto-optimal points in the space of variants can be expressed as follows:

"A given variant is Pareto-optimal if none of its grades can be improved without deteriorating at least one of the others."

The set of non-dominated solutions from the entire

allowable search space is called the optimal set in the Pareto sense, and these solutions form the so-called Pareto front, solutions from this set are not dominated by any other, so in this sense they are optimal solutions for multi-criteria optimization problem.

Because non-Pareto-optimal variants can be improved for all criteria, the introduction of the Pareto-optimal concept has reduced the problem of finding a solution to a task with multiple criteria for selecting a point from this set.

However, the question remains whether we can clearly determine which Pareto-optimal point is the best. Philosophers and practitioners have worked on the answer to this question for many years.

There are the following methods to solve this task:

- Bentham's utilitarianism principle,
- Rawls' principles of justice,
- Salukvadze reference point,
- Benson weighted sum method,
- Haimes  $\epsilon$ -restrictions method,
- Goal programming method (Fig. 7).

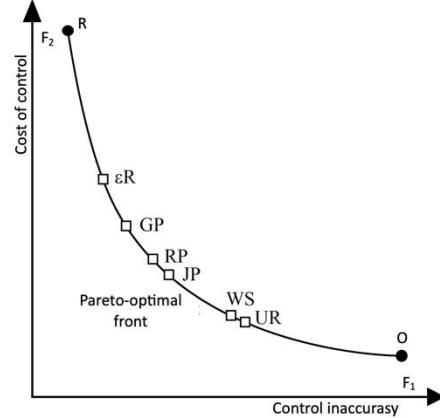


Fig. 7. Comparison of multi-criteria static optimization methods of the vessel traffic control process at the reference course: R – Restricted area,  $\epsilon$ R –  $\epsilon$ -Restrictions, GP – Goal Programming, RP – Reference Point, JP – Justice Principle, WS – Weighted Sum, UR – Utilitarianism Rule, O – Open sea

### IV.2. Dynamic multi-criteria optimization

The task of dynamic optimization of the multi-criteria quality indicator  $F$  can be presented as follows:

$$\min F = \min \sum_{i=1}^K f_{oi}(x, u, t), i = 1, 2, \dots, n; k = 1, 2, \dots, K \quad (20)$$

One of the more commonly used methods is the weighted sum of the integral function:

$$\sum_{k=1}^K f_{ok}(x, u, t) = \lambda_1 f_{o1} + \lambda_2 f_{o2} + \dots + \lambda_K f_{oK} \quad (21)$$

An example is the optimal stabilization of the ship's course using the optimal autopilot with the following control criterion:

$$\min F = \min \frac{1}{T_0} \int_t^{t+T_0} (\Delta\psi^2 + \lambda\alpha^2) dt \quad (22)$$

where:

$T_0$  - the time during which the ship passes the maximum distance on a given course,  
 $\Delta\psi$  - deviation of the actual course of the ship measured by the gyrocompass from the set value,  
 $\alpha$  - rudder angle,  
 $\lambda$  - weight coefficient, which is a trade-off between steering accuracy and control cost,  $\lambda \geq 0,1$  for ship navigation in restricted waters and  $\lambda \geq 10$  for ship navigation in open waters [13-16].

#### IV.3. Game multi-criteria optimization

Taking into account the form of a quality indicator, the issues of optimal control of transport or logistics processes can be divided into three groups, for which the cost of the process:

- is an explicit control function,
- depends on the control method and on some accidental event with a known statistical description,
- is determined by the choice of control method and some indeterminate factor with unknown statistical description.

The last group of issues concerns the quarterly transport or logistics processes whose synthesis is carried out using methods of game theory [17-19].

Taking into account the high complexity of the general differential game model become simplified models are formulated for practical synthesis of control algorithms, with the use of selected methods of artificial intelligence.

Appropriate algorithms for safe ship control in collision situations can be assigned to individual process models. Positional and matrix game models are used for the practical synthesis of control algorithms:

##### 1. Multi-stage positional game algorithm

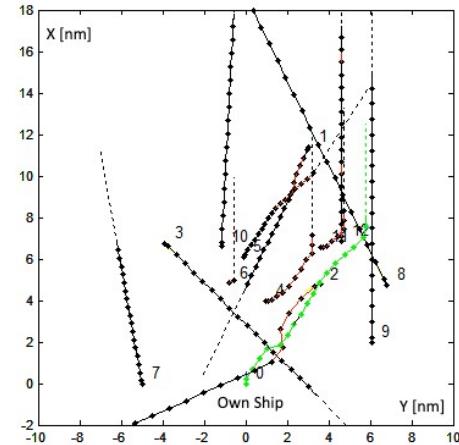


Fig. 8. Ship trajectories in non-cooperative positional game  $F$   
 $F^* = \text{minmaxminL}$ , when passing 12 encountered ships in good visibility at sea,  $L$  – distance to the nearest turning point on the reference trajectory

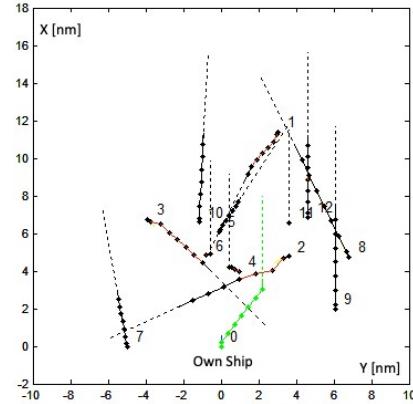


Fig. 9. Ship trajectories in cooperative positional game  
 $F^* = \text{minminminL}$ , when passing 12 encountered ships in good visibility at sea

##### 2. Multi-step matrix game algorithm

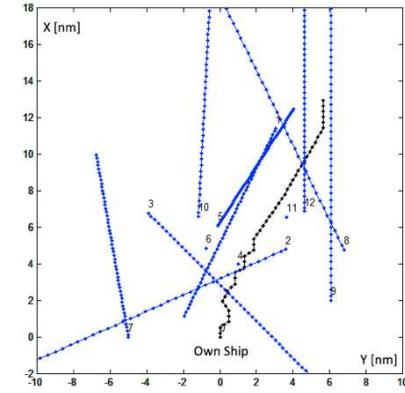


Fig. 10. Ship trajectories in non-cooperative matrix game  
 $F^* = \text{minmax } r$ , when passing 12 encountered ships in good visibility at sea,  $r$  – risk of collision

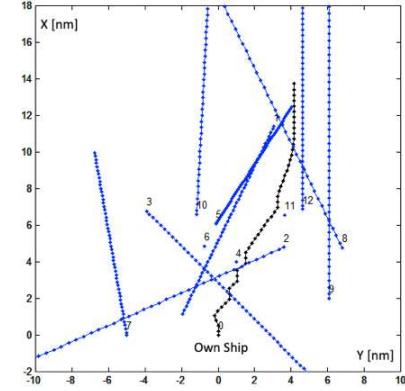


Fig. 11. Ship trajectories in cooperative matrix game  
 $F^* = \text{minmin } r$ , when passing 12 encountered ships in good visibility at sea

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## V. CONCLUSIONS

In synthesis of the controller or the optimal control algorithm for a given transport or logistic object, both static and dynamic analytical and numerical optimization methods can be used.

However, various optimization tasks in practical applications are most often solved by means of appropriate numerical methods of static and dynamic optimization.

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W artykule przedstawiono cztery główne rozdział, które pozwalają sformułować zadanie optymalizacji i wybrać metodę jego rozwiązania, od metod optymalizacji statycznej i dynamicznej do optymalizacji jedno i wielokryterialnej. W grupie metod optymalizacji statycznej metody te są bez ograniczeń i z ograniczeniami, gradientowe i bez gradientowe oraz heurystyczne. Metody optymalizacji dynamicznej dzielą się na podstawowe - bezpośrednie i pośrednie oraz specjalne. Szczególną uwagę zwrócono na optymalizację wielokryterialną w podejściu do jednego obiektu jako optymalizację statyczną i dynamiczną oraz optymalizację wielu obiektów w scenariuszach sterowania rozgrywającego. Artykuł pokazuje nie tylko klasyczne metody optymalizacji, które zostały opracowane wiele lat temu, ale także najnowsze w tej dziedzinie, w tym między innymi metody roju cząstek.

**Słowa kluczowe:** metody optymalizacji, sterowanie optymalne, bezpieczne sterowanie statkiem, symulacja komputerowa

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